Group Interaction in Research and the Use of General Nesting Spatial Models[*]

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Abstract

This paper tests the feasibility and empirical implications of a spatial econometric model with a full set of interaction effects and weight matrix defined as an equally weighted group interaction matrix applied to research productivity of individuals. We also elaborate two extensions of this model, namely with group fixed effects and with heteroskedasticity. In our setting the model with a full set of interaction effects is overparameterised: only the SDM and SDEM specifications produce acceptable results. They imply comparable spillover effects, but by applying a Bayesian approach taken from LeSage (2014), we are able to show that the SDEM specification is more appropriate and thus that colleague interaction effects work through observed and unobserved exogenous characteristics common to researchers within a group.

Keywords: Spatial econometrics, identification, heteroskedasticity, group fixed effects, interaction effects, research productivity

JEL Classification: C21, D85, I23, J24

1 Introduction

For reasons to be identified in this paper, a linear spatial econometric model with a full set of interaction effects, namely among the dependent variable, the exogenous variables, and among the disturbances, is almost never used in empirical applications. The introductory textbook in spatial econometrics by LeSage and Pace (2009) illustrates this. In their overview of spatial econometric models, they duly consider all extensions of the linear regression model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$ in which $\mathbf{X}$ is exogenous and ϵ is an *i.i.d.* disturbance, except the model with a full set of interaction effects. The spatial autoregressive (SAR) model contains a spatially lagged dependent variable $\mathbf{WY}$, where the symbol $\mathbf{W}$ represents the weights matrix arising from the spatial arrangement of the geographical units in the sample. The spatial error model (SEM) contains a spatially autocorrelated disturbance, $\mathbf{U}$, usually constructed via the spatial autoregression, $\mathbf{U} = \lambda \mathbf{WU} + \epsilon$. The model with both a spatially lagged dependent variable, $\mathbf{WY}$, and a spatially autocorrelated disturbance, $\mathbf{WU}$, is denoted by the term SAC in LeSage and Pace (2009, p.32), though this acronym is not explained.[1] The spatial lag of $\mathbf{X}$ model (SLX) contains spatially lagged exogenous variables, $\mathbf{WX}$; the spatial Durbin model (SDM) a spatially lagged dependent variable and spatially lagged exogenous variables, $\mathbf{WY}$ and $\mathbf{WX}$; and the spatial Durbin error model (SDEM) spatially lagged exogenous variables and a spatially autocorrelated error term, $\mathbf{WX}$ and $\mathbf{WU}$. The model with a spatially lagged dependent variable, spatially lagged exogenous variables, and a spatially autocorrelated disturbance is in fact mentioned, namely on page 53, but not taken seriously to judge from the fact that all equations in the book are numbered, except this one.

Part of the motivation for this paper is to take the opportunity to challenge two popular misconceptions about models of this type that have arisen in spatial econometrics. The first of these erroneous views holds that the parameters of a linear regression model specified to include interaction effects among the dependent variable, the exogenous variables, and among the disturbances cannot be identified. A possible cause of this mistake could be a loose reading of Manski (1993) who demonstrated the failure of identification in an equation in which the endogenous peer effect was assumed to operate via the group means of the dependent variable, labeling his result "the reflection problem". The second misconception goes back to Anselin and Bera (1998), according to whom an additional

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[1] Elhorst (2010) labels this model the Kelejian-Prucha model after their article in 1998 since they are the first to set out an estimation strategy for this model, also when the spatial weights matrix used to specify the spatial lag and the spatial error is the same. Kelejian and Prucha themselves alternately use the terms SARAR or Cliff-Ord type spatial model.

identification requirement when applying ML estimators is that the spatial weights matrix of the spatially lagged dependent variable must be different from the spatial weights matrix of the spatially autocorrelated disturbance, though without formally deriving this identification restriction, either in that study or any related work.

Lee, Liu and Lin (2010) are the first who provide formal proofs and conditions under which the parameters of a linear regression model specified with interaction effects among the dependent variable, among the exogenous variables, and among the disturbances are identified. Importantly, their proofs are limited to a spatial weights matrix that is specified as an equally weighted group interaction matrix with a zero diagonal. This is a block diagonal matrix where each block represents a group of units that interact with each other but not with members of other groups. In that case the value of all off-diagonal elements within a block equals $w_{ij} = 1/(n_r - 1)$, where n_r denotes the number of units in group r. Despite the fact that such a group interaction matrix is not very popular in applied spatial econometric research, Lee, Liu and Lin's findings make clear that Manski's reflection problem does not carry over to the case in which the endogenous peer effect operates via the mean of each individual's peers, since this mean is different for each individual, and that Anselin and Bera's (1998) identification restriction is unnecessary.

On the other hand, notice that the difference between this form of interaction matrix and the "group mean" version that leads to Manski's reflection problem can be very small: in the latter, the matrix would not have a zero diagonal, each element being equal to $w_{ij}^* = 1/n_r$. Furthermore, as Lee, Liu and Lin (2010, p.156) note, if the groups are large, identification will be weak. This problem may worsen if group fixed effects are included, which Lee, Liu and Lin (2010) put forward as an important model extension. In a footnote, they (ibid, p.147) motivate this extension as a first step towards capturing endogenous group formation. Moreover, back in 1988, Anselin (1988, pp. 61-65) advocated a "General model" with all types of interaction effects and heteroskedastic disturbances, though without providing conditions under which the parameters of this model are identified. Lee, Liu and Lin (2010) establish identification for a model in which the spatial weights matrix has a group interaction form, by introducing explicit rank conditions. The parameters of Anselin's general model will be identified under an extended set of similar such conditions, the function of which is primarily to rule out rogue special cases. Some of these conditions are discussed in Section 2.1, but in this paper our main purpose is to address the empirical usefulness of the heteroskedastic counterpart of the model in Lee, Liu and Lin (2010), since this turns out to be strongly supported by the data.

Altogether, then, we aim to test the feasibility, empirical implications and relevance of

a group interaction model with a full set of interaction effects, as well as the extensions with group fixed effects as proposed in Lee, Liu and Lin (2010) and heteroskedastic disturbances as proposed in Anselin (1988). We designate these models as the General Nesting Spatial (GNS) model, the Group Fixed Effects GNS (GFE-GNS) model, and the Heteroskedastic GNS (HGNS) model. For this purpose we use data that encompass all scientists employed at economics, business, and finance departments of 83 universities in Austria, Germany and German speaking Switzerland to identify the type and to estimate the extent of research interactions among colleagues within a university on individual research productivity.

Our findings throw new light on the seminal works of Anselin (1988), Anselin and Bera (1998), LeSage and Pace (2009), Lee, Liu and Lin (2010), and many empirical studies adopting one or more of the models explained in these works. Firstly, in our setting the well-known SAR, SEM, SLX and SAC models demonstrably lead to incorrect inferences based on the direct and indirect effects estimates that can be derived from the point estimates of the different models. Interestingly, the group interaction model is one of the few models for which convenient explicit expressions for these direct and indirect effects estimates can be derived, as we will show. Secondly, the GNS model appears to be over-parameterised; the significance of the coefficient estimates in this model is lower than in the nested SDM and SDEM models. Thirdly, only the SDM and SDEM specifications produce acceptable results. Apparently, in our case, interaction effects among both the dependent variable and the error terms do not perform well together, though not for reasons of identification as suggested by Anselin and Bera (1998) but for reasons of over-fitting. Fourthly, the extension with group fixed effects appears to have little empirical relevance. This is due to high correlation between the $\mathbf{X}$ and the $\mathbf{WX}$ variables that arises after transformation by group-demeaning, as we will show both mathematically and empirically. By contrast, the extension with heteroskedasticity appears to have more empirical relevance, bringing us back to the seminal work of Anselin (1988). Finally, the findings of our empirical application show that the kind and magnitude of interaction effects driving research productivity of scientific communities are in line with previous studies on peer effects in academia using a natural experiment setting (Waldinger 2011, Borjas and Doran 2014).

The remainder of this paper is organized as follows. Section 2 sets out the GNS model, its basic properties, and the two extensions. Section 3 describes the Matlab routines to find the optimum of the log-likelihood function. After a description of our data, our measure of research productivity, and its potential determinants in Section 4, Section 5

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reports and reviews the results of our empirical analysis. The paper concludes with a summary of the main results in Section 6.

2 The GNS model and its extensions

The model with both group specific effects and heteroskedastic disturbances is closely related to those treated by Anselin (1988), Bramoullé, Djebbari and Fortin (2009), and Lee, Liu and Lin (2010). This model can be viewed either as a generalisation of the "General Model" in Anselin (1988) with group specific effects, restricted here to the group interaction setting, or as a generalisation of the group interaction model of Lee, Liu and Lin (2010) expanded to allow for heteroskedastic disturbances. In notation that adapts Anselin's to the group interaction setting of Lee, Liu and Lin (2010), the extended GNS model is, for the rth group:

$$
\begin{aligned}
\mathbf{Y}_r &= \rho_0 \mathbf{W}_r \mathbf{Y}_r + \mathbf{1}_{n_r} \delta_{r0} + \mathbf{X}_r \beta_0 + \mathbf{Q}_r \mathbf{X}_r \gamma_0 + \mathbf{U}_r \qquad\qquad (1)\\
\mathbf{U}_r &= \lambda_0 \mathbf{M}_r \mathbf{U}_r + \epsilon_r \\
&\quad E\{\epsilon_r\} = \mathbf{0}_{n_r}, \qquad E\{\epsilon_r \epsilon_r^{'}\} = \mathbf{\Omega}_r \\
\omega_{r,ii} &= h_r(\alpha_0, \mathbf{Z}_{r,i}) > 0, \quad \omega_{r,ij} = 0, \quad i \neq j, \quad i,j = 1, ..., n_r \\
r &= 1, ..., \bar{r}
\end{aligned}
$$

where n_r is the size of the rth group, $\bar{r}$ is the number of groups, $\mathbf{1}_{n_r} = [1, 1, ..., 1]'$ is an $n_r \times 1$ vector, $[\mathbf{1}_{n_r} \vdots \mathbf{X}_r \vdots \mathbf{Q}_r \mathbf{X}_r]$ is a matrix of n_r rows with full column rank with elements that are independent of the shocks, ϵ_r, and $\mathbf{Y}_r$ is an $n_r \times 1$ vector of observations of the dependent variable, and $\omega_{r,ii}$ is an element of the $n_r \times n_r$ matrix $\mathbf{\Omega}_r$. When the group fixed effects, $\mathbf{1}_{n_r} \delta_{r0}$, are absent, they are replaced by a single intercept common to all groups, $\mathbf{1}_{n_r} \delta_0$. The inclusion of group-specific fixed effects, as in Lee, Liu and Lin (2010), requires the model to be transformed to avoid the incidental parameter problem, while also ruling out the estimation of the effects of exogenous covariates that are constant within groups. For this reason, it seems appropriate to separate these two cases when discussing this extension. We start with the model without group-specific fixed effects, and then consider within-group interactions in the disturbance. The $n_r \times n_r$ matrices of non-negative constants, $\mathbf{W}_r, \mathbf{Q}_r$, and $\mathbf{M}_r$ are of the form $\mathbf{W}_r = \mathbf{Q}_r = \mathbf{M}_r = \frac{1}{n_r - 1}[\ \mathbf{1}_{n_r} \mathbf{1}_{n_r}^{'} - \mathbf{I}_{n_r}]$, as in Lee, Liu and Lin (2010). It will be assumed that the matrices $[\mathbf{I}_{n_r} - \rho_0 \mathbf{W}_r] = \mathbf{A}_r$ and $[\mathbf{I}_{n_r} - \lambda_0 \mathbf{M}_r] = \mathbf{B}_r$ are non-singular with inverses as given later in the paper. Further, it is assumed that

there is no redundancy in the parameters - that is, there is no common factor restriction relating β_0, γ_0 and ρ_0 of the form discussed by Lee, Liu and Lin (2010, p.153).

The variables, $\mathbf{Z}_{r,i}$ that determine the pattern of heteroskedasticity are assumed to be observed without error, while the associated parameters, α, must be estimated. In our application, $\mathbf{Z}_{r,i} = [1, n_r]$ and $h_r(\alpha_0, \mathbf{Z}_{r,i}) = \alpha_{01} + \alpha_{02}n_r$ so that the disturbances have variance proportional to group size. In the homoskedastic model $\alpha_{02} = 0$ and $\alpha_{01} = \sigma^2$, which yields $\mathbf{\Omega} = \sigma^2\mathbf{I}$. Since only university-specific and not reseacher-specific determinants are used to model heteroskedasticity, the subscript i of $\mathbf{Z}_{r,i}$ may also be dropped. The Normal likelihood, first-order conditions and information matrix corresponding to (1), for the homoskedastic case are set out in Lee, Liu and Lin (2010, p. 151), and for the heteroskedastic case without group fixed effects in Anselin (1988, pp. 61-65). These models can be estimated by ML or QML. In the first case, the disturbances are assumed to be Normally distributed. In the second case, it is required that some absolute moment higher than the 4th exists.

2.1 Case 1: no group-specific fixed effects

Write $N = \sum_{r=1}^{\bar{r}} n_r$ for the total sample size, and $\mathbf{W}$, $\mathbf{Q}$, $\mathbf{M}$, $\mathbf{\Omega}$, $\mathbf{A}$ and $\mathbf{B}$ for the $N \times N$ block-diagonal matrices with diagonal blocks given by $\mathbf{W}_r$, $\mathbf{Q}_r$, and so on, for $r = 1, ..., \bar{r}$ and similarly $\mathbf{X}$ for the matrix of exogenous regressors. For convenience, write the full parameter vector as $\theta_0 = (\delta_0, \beta_0', \gamma_0', \lambda_0, \rho_0, \alpha_0')' = (\beta_0^{*'}, \lambda_0, \rho_0, \alpha_0')'$ and suppose it is an interior point in the compact space $\mathcal{T}$. Then, writing $\mathbf{X}^* = (\mathbf{1} \vdots \mathbf{X} \vdots \mathbf{QX})$ so that the exogenous part of the mean function of the model can be written compactly as $\mathbf{X}^*\beta_0^*$ and writing $\eta = \mathbf{\Omega}^{-1/2}\epsilon$ so that the $N-$ dimensional random vector η has mean $\mathbf{0}$ and covariance matrix $\mathbf{I}_N$, the Normal log-likelihood takes the form

$$l(\mathbf{Y}, \mathbf{X}, \mathbf{W}, \mathbf{Q}, \mathbf{M}, \theta) = -\frac{N}{2}\ln 2\pi - \frac{1}{2}\ln|\mathbf{\Omega}| + \ln||\mathbf{A}|| + \ln||\mathbf{B}|| - \frac{1}{2}\eta'\eta \qquad (2)$$

in which the sum of squares term is

$$\eta'\eta = \epsilon'\mathbf{\Omega}^{-1}\epsilon$$
$$\text{where } \epsilon = \mathbf{B}(\mathbf{AY} - \mathbf{X}^*\beta_0^*) = \mathbf{BU}.$$

It follows that for given $(\tilde{\lambda}, \tilde{\rho}, \tilde{\alpha}')$ the ML estimator of β^* when it exists, is given by GLS as

$$\tilde{\beta}^* = (\mathbf{X}^{*'}\tilde{\mathbf{B}}'\tilde{\mathbf{\Omega}}^{-1}\tilde{\mathbf{B}}\mathbf{X}^*)^{-1}\mathbf{X}^{*'}\tilde{\mathbf{B}}'\tilde{\mathbf{\Omega}}^{-1}\tilde{\mathbf{B}}\tilde{\mathbf{A}}\mathbf{Y}. \tag{3}$$

In the homoskedastic model, we have $\mathbf{\Omega} = \sigma^2 \mathbf{I}$, as a result of which the matrix $\tilde{\mathbf{\Omega}}$ drops out of (3). Consequently, the variance parameter σ^2 can be solved from its first-order maximizing condition and its solution substituted in the log-likelihood function. In the heteroskedastic case, the first-order maximizing conditions do not give a closed form solution for $\tilde{\alpha}$ in terms of the residual vector associated with (3), $\tilde{\epsilon}(\tilde{\lambda}, \tilde{\alpha}, \tilde{\rho})$. Nevertheless, concentration with respect to β^* remains helpful both computationally and analytically. The concentrated log-likelihood function of (ρ, λ, α) is

$$\ln L(\rho, \lambda, \alpha) = -\frac{N}{2}\ln 2\pi - \frac{1}{2}\ln|\mathbf{\Omega}| + \ln||\mathbf{A}|| + \ln||\mathbf{B}|| - \frac{1}{2}\tilde{\epsilon}'\mathbf{\Omega}^{-1}\tilde{\epsilon} \tag{4}$$

Lee, Liu and Lin (2010) make the following assumptions to prove consistency of the (Q)ML estimator of the parameters in this model. Each group, r, is of fixed size, n_r, and upper bounded. This implies that the sample can only grow without limit by the addition of more groups, that is, as $\bar{r} \to \infty$. In addition, these groups should be of different sizes, a condition that is also required for consistent estimation of α. It is possible, though laborious, to show directly via the rank of the relevant sub-matrix of the information matrix $\mathcal{I}(\theta)$ that in the case $\bar{r} = 2$, α is identified provided $n_1 \neq n_2$. The matrix, $\mathbf{X}^{*'}\mathbf{B}'\mathbf{\Omega}^{-1}\mathbf{B}\mathbf{X}^*$ has full rank, and $\lim_{\bar{r}\to\infty}\{\frac{1}{\bar{r}}\mathbf{X}^{*'}\mathbf{B}'\mathbf{\Omega}^{-1}\mathbf{B}\mathbf{X}^*\}$ exists and is non-singular. These conditions require boundedness of the row and column sums of the weight matrices $\mathbf{W}_r$ and of the inverses $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$, each of which is automatically satisfied by the normalised weights assumed above. Lee (2007) derives additional conditions that need to be satisfied in case the spatial weights matrix is not row-normalized. The rank condition for identification of β^* also implies that the columns of $\mathbf{X}$ and $\mathbf{QX}$ must not be collinear if both are to have non-zero coefficients; by considering the case, $\bar{r} = 2$, and assuming $n_1 \neq n_2$ it can be shown that any such covariates must vary over the members of at least one of the groups. However, the rank and existence conditions just stated cover such cases.

Further, Lee, Liu and Lin (2010) deal with the need to bound linear and quadratic forms involving the exogenous regressors by treating these as fixed constants, remarking that this is just a matter of convenience (Lee, Liu and Lin 2010, footnote 16) and would be easily generalised to include stochastic regressors; hence we just repeat this assumption here.

Finally, they assume the shocks are *i.i.d.* with zero mean, constant variance, and that some absolute moment higher than the 4th exists. This last can be modified to suit the heteroskedastic case, perhaps most simply by assuming an underlying *i.i.d.* random variable with mean zero and unit variance and enough higher moments that is simply scaled up by the required non-stochastic function, i.e. by $(\alpha_{01}+\alpha_{02}n_r)^{1/2}$. If the underlying variable is Normally distributed, then the limiting covariance matrix of $\tilde{\theta}$ coincides with the limit of the inverse of the information matrix; if not, then a correction matrix involving 3rd and 4th moments is required. We now focus below on ML estimation of the different models.

2.2 Case 2: including group-specific fixed effects

If the group intercepts, δ_{r0}, vary across groups $r = 1, ..., \bar{r}$, the data must be transformed to avoid the growth in the number of parameters with sample size, the so-called incidental parameter problem. Lee, Liu and Lin (2010) solve this problem by introducing an orthonormal transformation, which they label by the matrix $\mathbf{F}$. However, by closer inspection of $\mathbf{F}$, we show below that an acute problem of multicollinearity is likely to be induced by its use.

Because of the very simple form of the group interaction matrices in the present case, the group fixed effects could be also eliminated by deviation from the group means as in a standard panel data model. However, as this would induce dependence in the transformed disturbances, Lee, Liu and Lin (2010) use the alternative $\mathbf{F}$ transformation. This transformation decreases the number of observations by one for each group r. Let $\mathbf{J}_{n_r}$ denote the deviation from group mean operator for group r, i.e. $\mathbf{J}_{n_r} = [\mathbf{I}_{n_r} - n_r^{-1}\mathbf{1}_{n_r}\mathbf{1}'_{n_r}]$, and introduce the orthonormal decomposition, $(\mathbf{F}_{n_r}, \mathbf{1}_{n_r}/\sqrt{n_r})$ such that $\mathbf{J}_{n_r} = \mathbf{F}_{n_r}\mathbf{F}'_{n_r}$, $\mathbf{F}'_{n_r}\mathbf{F}_{n_r} = \mathbf{I}_{n_r-1}$ and $\mathbf{F}'_{n_r}\mathbf{1}_{n_r} = \mathbf{0}_{n_r-1}$. An explicit solution for the $n_r \times (n_r - 1)$ matrix $\mathbf{F}_{n_r}$ is easily seen to be

$$
\mathbf{F}_{n_r} = \begin{bmatrix}
0 & 0 & \cdots & 0 & -\sqrt{\frac{n_r-1}{n_r}} \\
\vdots & \vdots & & -\sqrt{\frac{n_r-2}{n_r-1}} & \sqrt{\frac{1}{n_r(n_r-1)}} \\
\vdots & 0 & & \sqrt{\frac{1}{(n_r-1)(n_r-2)}} & \vdots \\
0 & -\sqrt{\frac{2}{3}} & & \vdots & \vdots \\
-\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{6}} & & \vdots & \vdots \\
\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{6}} & \cdots & \sqrt{\frac{1}{(n_r-1)(n_r-2)}} & \sqrt{\frac{1}{n_r(n_r-1)}}
\end{bmatrix}. \tag{5}
$$

To exploit this transformation, observe that because $\mathbf{F}'_{n_r}\mathbf{1}_{n_r} = \mathbf{0}$ it follows that $\mathbf{F}'_{n_r}\mathbf{B}_r = (1 + \frac{\lambda_0}{n_r - 1})\mathbf{F}'_{n_r}$ and similarly $\mathbf{F}'_{n_r}\mathbf{A}_r = (1 + \frac{\rho_0}{n_r - 1})\mathbf{F}'_{n_r}$ so that the relation

$$\epsilon_r = \mathbf{B}_r(\mathbf{A}_r\mathbf{Y}_r - \mathbf{X}_r^*\beta_0^*) \tag{6}$$

transforms to

$$
\begin{aligned}
\mathbf{F}'_{n_r}\epsilon_r &= \mathbf{F}'_{n_r}\mathbf{B}_r(\mathbf{A}_r\mathbf{Y}_r - \mathbf{X}_r^*\beta_0^*) \\
&= \left(1 + \frac{\lambda_0}{n_r - 1}\right)\mathbf{F}'_{n_r}(\mathbf{A}_r\mathbf{Y}_r - \mathbf{X}_r^*\beta_0^*) \\
&= \left(1 + \frac{\lambda_0}{n_r - 1}\right)\left(1 + \frac{\rho_0}{n_r - 1}\right)\mathbf{F}'_{n_r}\mathbf{Y}_r - \left(1 + \frac{\lambda_0}{n_r - 1}\right)\mathbf{F}'_{n_r}\mathbf{X}_r^*\beta_0^*.
\end{aligned}
\tag{7}
$$

Defining the transformed objects, $\mathbf{Y}_r^* = \mathbf{F}'_{n_r}\mathbf{Y}_r$, $\mathbf{X}_r^{**} = \mathbf{F}'_{n_r}\mathbf{X}_r^*$, together with β_0^{**} being β_0^* with the fixed effect removed, then we obtain the transformed structure, without group fixed effects

$$\left(1 + \frac{\lambda_0}{n_r - 1}\right)\left(1 + \frac{\rho_0}{n_r - 1}\right)\mathbf{Y}_r^* - \left(1 + \frac{\lambda_0}{n_r - 1}\right)\mathbf{X}_r^{**}\beta_0^{**} = \epsilon_r^* \text{ say.} \tag{8}$$

Here, the rth block is of dimension $n_r - 1$, and $E\{\epsilon_r^*\epsilon_r^{*'}\} = \mathbf{I}_{n_r-1}(\alpha_{01} + \alpha_{02}n_r)$. Note that the decrease in the number of observations by one in each group is merely a reduction in the number of degrees of freedom, since the information of all n_r observations in each group is still implied in the data. Further note the simplicity of (8). Interestingly, Lee, Liu and Lin (2010) do not write the transformed model in this simple form, introducing transformed versions of $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{W}$ instead (see their 3.3 and 3.4). With suitable redefinitions we may thus write the model for the entire transformed sample as

$$\mathbf{B}^*[\mathbf{A}^*\mathbf{Y}^* - \mathbf{X}^{**}\beta_0^{**}] = \epsilon^* \tag{9}$$

in which $\mathbf{B}^*$ and $\mathbf{A}^*$ are defined in terms of a transformed weight matrix, $\mathbf{W}^*$ say.

However, since

$$\mathbf{A}^* = (\mathbf{I} - \rho_0\mathbf{W}^*), \tag{10}$$

$\mathbf{W}^*$ matches (8) only if it has diagonal blocks of the form

$$\mathbf{W}_r^* = \frac{-1}{n_r - 1}\mathbf{I}_{n_r-1} \tag{11}$$

and zeros everywhere else, giving an object that is much easier to interpret. From (11)

it immediately follows that $Tr\{\mathbf{W}_r^*\} = -1$ and that all its eigenvalues are $\frac{-1}{n_r-1}$. This implies that the eigenvalues of $\mathbf{W}^*$ are $\bar{r}$ sets of $\frac{-1}{n_r-1}$ each with multiplicity $(n_r - 1)$. Furthermore, except for the eigenvalues of $\mathbf{W}$ that are identical to those of $\mathbf{W}^*$, it follows that $\mathbf{W}$ has $\bar{r}$ additional eigenvalues of 1, one for each group r.

Using the results of the $\mathbf{F}$-transformation, we now demonstrate that in our setting a model with group fixed effects and spatially lagged exogenous variables, $\mathbf{WX}$, encounters near multicollinearity. Consider the first expression in equation (1)

$$\mathbf{Y}_r = \rho_0 \mathbf{W}_r \mathbf{Y}_r + \mathbf{1}_{n_r}\delta_{r0} + \mathbf{X}_r\beta_0 + \mathbf{W}_r\mathbf{X}_r\gamma_0 + \mathbf{U}_r \quad \text{with} \quad \mathbf{W}_r = \frac{1}{n_r-1}(\mathbf{1}_{n_r}\mathbf{1}'_{n_r} - \mathbf{I}_{n_r}).$$

In this model the inclusion of all the group intercept terms would give the same coefficients on everything else as we obtain by first subtracting all the group means from $\mathbf{Y}_r$, $\mathbf{X}_r$ and $\mathbf{WX}_r$ by multiplication by $\mathbf{J}_{n_r} = [\mathbf{I}_{n_r} - n_r^{-1}\mathbf{1}_{n_r}\mathbf{1}'_{n_r}]$. Consequently, after transformation by group de-meaning we obtain a set of columns each with blocks of entries of the form $(\mathbf{I}_{n_r} - \frac{1}{n_r}\mathbf{1}_{n_r}\mathbf{1}'_{n_r})\mathbf{X}_r$ and similarly a second set with blocks of the form $(\mathbf{I}_{n_r} - \frac{1}{n_r}\mathbf{1}_{n_r}\mathbf{1}'_{n_r})\mathbf{W}_r\mathbf{X}_r$. However, since

$$
\begin{aligned}
\left(\mathbf{I}_{n_r} - \frac{1}{n_r}\mathbf{1}_{n_r}\mathbf{1}'_{n_r}\right)\mathbf{W}_r &= \left(\mathbf{I}_{n_r} - \frac{1}{n_r}\mathbf{1}_{n_r}\mathbf{1}'_{n_r}\right)\frac{1}{n_r-1}\left(\mathbf{1}_{n_r}\mathbf{1}'_{n_r} - \mathbf{I}_{n_r}\right) \qquad (12)\\
&= \frac{-1}{n_r-1}\left(\mathbf{I}_{n_r} - \frac{1}{n_r}\mathbf{1}_{n_r}\mathbf{1}'_{n_r}\right) + \frac{1}{n_r-1}\left(\mathbf{I}_{n_r} - \frac{1}{n_r}\mathbf{1}_{n_r}\mathbf{1}'_{n_r}\right)\mathbf{1}_{n_r}\mathbf{1}'_{n_r}\\
&= \frac{-1}{n_r-1}\left(\mathbf{I}_{n_r} - \frac{1}{n_r}\mathbf{1}_{n_r}\mathbf{1}'_{n_r}\right)
\end{aligned}
$$

the second set of transformed variables obtained by transforming $\mathbf{W}_r\mathbf{X}_r$ are only different from the first set obtained by transforming $\mathbf{X}_r$ by virtue of the leading $\frac{-1}{n_r-1}$ terms. This implies that they would be perfectly collinear if all the groups were the same size. However, also if group sizes differ, they are most likely to be near collinear. In Section 5 we show that the degree of multicollinearity in our empirical analysis is indeed rather high; we find values up to 0.99. In other words, while the parameters of the GFE-GNS model might be formally identified under the conditions summarized above, the case of near multicollinearity will create statistical problems in that the parameter estimates are imprecise.

2.3 Direct and indirect effects in the case without group fixed effects

In our application $\mathbf{Q}_r = \mathbf{W}_r$, thus the reduced form of the model (1) with $\bar{r}$ groups is

$$\mathbf{Y} = (\mathbf{I}_N - \rho_0 \mathbf{W})^{-1}\{\mathbf{1}_N \delta_0 + \mathbf{X}\beta_0 + \mathbf{W}\mathbf{X}\gamma_0 + \mathbf{U}\}.$$

We obtain the direct and indirect (spillover) effects from the above equation building on the assumption that $\mathbf{X}$ is independent of $\mathbf{U}$ and therefore causally predetermined with respect to $\mathbf{Y}$. Following LeSage and Pace (2009), the direct effect is calculated as the average diagonal element of the matrix $(I_N - \rho_0 \mathbf{W})^{-1}\{\mathbf{I_N}\beta_0 + \mathbf{W}\gamma_0\}$, and the indirect effect as the average row or column sum of the off-diagonal elements of that matrix.

Because of the group structure, the matrix $(\mathbf{I}_N - \rho_0 \mathbf{W})^{-1}$ is block-diagonal, composed of $\bar{r}$ blocks, the rth having dimension n_r, the number of individuals in the rth group. In addition, the inverse of each block is known to be

$$(\mathbf{I}_{n_r} - \rho_0 \mathbf{W}_r)^{-1} = \left(\frac{n_r - 1}{n_r - 1 + \rho_0}\right)\left[\mathbf{I}_{n_r} + \left(\frac{\rho_0}{(n_r - 1)(1 - \rho_0)}\right)\mathbf{1}_{n_r}\mathbf{1}'_{n_r}\right]. \qquad (13)$$

As a result, the direct and indirect effects are associated with each of the blocks (i.e. each group has potentially different effects). For group r the direct effect has two components, being the sum of a typical diagonal element of $(\mathbf{I}_{n_r} - \rho_0 \mathbf{W}_r)^{-1}$ scaled by β_0 and a typical diagonal element of $(\mathbf{I}_{n_r} - \rho_0 \mathbf{W}_r)^{-1}\mathbf{W}_r$ scaled by γ_0. Similarly, the indirect effects have two components, one obtained by summing the off-diagonal entries of a typical column of $(\mathbf{I}_{n_r} - \rho_0 \mathbf{W}_r)^{-1}$ scaled by β_0 and the other by summing the off-diagonal entries of a typical column of $(\mathbf{I}_{n_r} - \rho_0 \mathbf{W}_r)^{-1}\mathbf{W}_r$ scaled by γ_0.

By inspection a typical diagonal entry of $(\mathbf{I}_{n_r} - \rho_0 \mathbf{W}_r)^{-1}$ is

$$\left(\frac{n_r - 1}{n_r - 1 + \rho_0}\right)\left[1 + \frac{\rho_0}{(n_r - 1)(1 - \rho_0)}\right] = \frac{n_r - 1 - \rho_0(n_r - 2)}{(n_r - 1 + \rho_0)(1 - \rho_0)} \equiv \mathrm{DE}_{\beta_0}(r) \qquad (14)$$

denoting the direct effect associated with β_0 in group r. Similarly, the typical off-diagonal entry, summed over a column, is

$$\left(\frac{n_r - 1}{n_r - 1 + \rho_0}\right)\frac{\rho_0(n_r - 1)}{(n_r - 1)(1 - \rho_0)} = \frac{(n_r - 1)\rho_0}{(n_r - 1 + \rho_0)(1 - \rho_0)} \equiv \mathrm{IE}_{\beta_0}(r). \qquad (15)$$

representing the indirect effect associated with β_0.

By writing $\boldsymbol{\Gamma}_r = \mathbf{1}_{n_r}\mathbf{1}'_{n_r}$, we have $\mathbf{W}_r = (n_r - 1)^{-1}(\boldsymbol{\Gamma}_r - \mathbf{I}_{n_r})$ and $\boldsymbol{\Gamma}_r^2 = n_r\boldsymbol{\Gamma}_r$, as a result of which

$$
\begin{aligned}
[\mathbf{I}_{n_r} - \rho_0\mathbf{W}_r]^{-1}\mathbf{W}_r &= \left(\frac{n_r - 1}{n_r - 1 + \rho_0}\right)\left[\mathbf{I}_{n_r} + \left(\frac{\rho_0}{(n_r - 1)(1 - \rho_0)}\right)\boldsymbol{\Gamma}_r\right]\mathbf{W}_r \qquad (16)\\
&= \left(\frac{1}{n_r - 1 + \rho_0}\right)\left[\mathbf{I}_{n_r} + \left(\frac{\rho_0}{(n_r - 1)(1 - \rho_0)}\right)\boldsymbol{\Gamma}_r\right](\boldsymbol{\Gamma}_r - \mathbf{I}_{n_r})\\
&= \left(\frac{1}{n_r - 1 + \rho_0}\right)\left[(1 - \rho_0)^{-1}\boldsymbol{\Gamma}_r - \mathbf{I}_{n_r}\right].
\end{aligned}
$$

By inspection the typical diagonal element of this matrix takes the form

$$
\left(\frac{1}{n_r - 1 + \rho_0}\right)\left[(1 - \rho_0)^{-1} - 1\right] = \frac{\rho_0}{(n_r - 1 + \rho_0)(1 - \rho_0)} \equiv \mathrm{DE}_{\gamma_0}(r) \qquad (17)
$$

which is the direct effect associated with γ_0. Similarly, the off-diagonal element, summed over a column

$$
\left(\frac{1}{n_r - 1 + \rho_0}\right)(1 - \rho_0)^{-1}(n_r - 1) = \frac{n_r - 1}{(n_r - 1 + \rho_0)(1 - \rho_0)} \equiv \mathrm{IE}_{\gamma_0}(r) \qquad (18)
$$

gives the indirect effect associated with γ_0. To obtain the direct and indirect effects over the whole sample, one should calculate the average over the r-different groups.

3 Estimation routines

To maximize the likelihood function (2) of the different general nesting models numerically, we developed routines building on previous work of LeSage (1999). LeSage provides a Matlab routine called "SAC" at his web site[2] that can be used to maximize the log-likelihood function of the homoskedastic general nesting model. Even if this routine was originally developed for estimating a SAC model, i.e a model with a spatially lagged dependent variable and a spatially autocorrelated error term, by computing the spatially lagged exogenous variables $\mathbf{WX}$ in advance and by specifying the argument $\mathbf{X}$ of this routine as $[\mathbf{X}\ \mathbf{WX}]$, it is also possible to obtain parameter estimates of the full model with homoskedastic errors. Since individual groups within our group interaction matrix $\mathbf{W}$ are relatively small and each group has its own set of characteristic roots, we also replaced the

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[2]www.spatial-econometrics.com

approximate calculation of $\log |\mathbf{I} - \rho_0 \mathbf{W}| + \log |\mathbf{I} - \lambda_0 \mathbf{W}|$ (see LeSage and Pace, 2009, Ch. 4) by the exact calculation $\sum_i \log(1 - \rho_0 \omega_i) + \sum_i \log(1 - \lambda_0 \omega_i)$, where ω_i $(i = 1, \ldots, n)$ denote the characteristic roots of the matrix $\mathbf{W}$ given below (11). Consequently, the calculation of the log determinants of the matrices $\mathbf{A}$ and $\mathbf{B}$ in the (concentrated) log-likelihood functions (2) and (4) produces more accurate results.[3] Finally, we also adapted this routine for heteroskedastic model specifications and for models with group fixed effects.

Since the coefficient vector β_0^* can be solved from the first-order conditions (Anselin 1988, equations 6.21-6.24), the log-likelihood function only needs to be maximized for the parameters ρ_0, λ_0 and α_0. An incidental advantage of the concentrated likelihood is reduced computation time. The standard errors and t-values of the parameter estimates are calculated from the asymptotic variance-covariance matrix following Anselin (1988, equations 6.25-6.34). The standard errors and t-values of the direct and indirect effects estimates are more difficult to determine, even though the analytical expressions of the direct and indirect effects are known (see equations 14-18). They depend on β_0, γ_0 and ρ_0 in a rather complicated way. To draw inferences regarding the statistical significance of the direct and indirect effects, we follow the suggestion of LeSage and Pace (2009, p. 39) and simulate the distribution of the direct and indirect effects using the variance-covariance matrix implied by the maximum likelihood estimates. If the full parameter vector θ is drawn D times from $N(\hat{\theta}, \text{AsyVar}(\hat{\theta}))$, the standard deviation of the estimated (in)direct effects is approximated by the standard deviation of the mean value of equations (14)-(18) over these D draws. We test the significance of our original ML (in)direct effects estimates using the corresponding simulated standard deviation.

4   Empirical illustration

For our empirical analysis we draw on a database that covers all researchers specializing in economics, business and finance employed at universities in German speaking countries.[4] For our purposes we extracted from this database all scientists beyond PhD level along with their journal publications released over the 1999-2008 period. To allow time for the youngest scholars' publications to appear, we included only those who graduated

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[3]We also improved two programming errors in the calculation of the variance-covariance matrix of the parameter estimates. The adapted SAC routine will be made available at http://www.regroningen.nl/elhorst/software.shtml or can be supplied on request.

[4]The database is under the auspices of the German Economic Association: www.socialpolitik. org. It is known across the German speaking region as the research monitoring database: www. forschungsmonitoring.org.

earlier than 2007. We excluded emeritus professors and academic staff involved only in administrative or teaching duties. Using these criteria, our data set contains 2580 researchers employed by 83 universities covering nearly the whole "space" of university research in economics, finance and business across the German speaking region.[5]

For each individual i in the data set, we measure the dependent variable, research productivity, as the researcher's average annual research productivity:

$$\text{Prod}_i = \frac{1}{y_i} \sum_{p_i=1}^{P_i} \frac{w_{p_i}}{a_{p_i}}. \tag{19}$$

This is the quality weighted sum of all journal articles of i, (P_i), published over the decade 1999-2008. Each article is divided by the corresponding number of coauthors a_{p_i}. The y_i is either the number of years since graduation or 10 if the graduation year goes back to more than 10 years. The weights w_{p_i} express the quality index of the journal in which the article was published.

For (19) we adopted the weighting scheme developed originally by the German business newspaper, Handelsblatt, which publishes individual and department rankings in economics and business administration across the German speaking countries. Handelsblatt uses distinct weighting schemes for economists and for researchers in finance and business administration. The scheme for economics is based on the so called CL-weights of EconLit journals by Combes and Linnemer (2003). Handelsblatt considers about 1200 journals, which are divided into 7 quality levels, ranging from 1 down to 0.05. The weighting scheme for finance and business administration includes only 761 journals and the journal quality is based on two sources: (i) the weighting scheme compiled by the German Academic Association for Business Administration[6], and (ii) the SSCI[7] impact factor. The two informations are then combined to assign each journal into one of the above 7 quality levels (Krapf 2011). Both schemes include international and German journals. Even though the weights of German journals tend to be small, they might be of some relevance for the career of German economists. Since our data set combines economists, finance and business researchers, our final individual productivity is a simple average of productivities (19) based on the two weighting schemes. To normalize for the skewed distribution of productivity—few researchers produce many articles and many

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[5]We dropped 14 universities with small economics and/or finance and business departments, losing only about 90 individuals.

[6]http://vhbonline.org/

[7]Social Sciences Citation Index

publish few or none—our dependent variable is then $\log(\text{Prod}_i + 1)$.

Our study uses the GNS model to estimate group effects. In this study, groups are represented by universities. Each researcher is considered to be a member of the university he or she was affiliated to at the end of 2009. Each individual's entire publication stock (1999-2008) is assigned via (19) to that particular university, even if the affiliation might have changed during that period. Combes and Linnemer (2003) label this productivity measure a "stock" measure and defend its use from the perspective of human capital currently embedded in a given university. The use of the stock measure also means that our GNS model reflects a steady-state equilibrium in distribution of human capital across groups.

The Lee, Liu and Lin (2010) identification condition (cf. Section 2) that groups should be of different sizes is readily fulfilled by the data. The department sizes of the 83 universities range from 10 to 160 with mean 31 and standard deviation of 23.

4.1 Determinants of research productivity

Economic theory describes the reward system in science as a collegiate reputation-based system and as such it functions well in satisfying efficiency in increasing the stock of reliable knowledge (Dasgupta and David 1994). Since reputation in science is strongly priority based, researchers race to be the first in publishing advances within their research fields. The best placed of this publication race are rewarded with top academic positions. The top positions allow these individuals to continue performing better than individuals employed at lower ranked institutions. The research output is thus marked by the advantage acquired in the early stage of somebody's career which cumulates over the life cycle. The concept of cumulative advantage is a basic feature of theoretical models of academic competition (e.g. Carayol 2008). The monetary reward in science consists of two components: a fixed salary and a bonus based on individual contributions to science. The non-monetary reward consists of the reward from puzzle solving and from recognition. In addition, research productivity is fed by individual inputs stemming from human capital formation, including age, cohort, and gender effects. Other individual inputs are time, cognitive abilities, knowledge base, extent of collaboration, and access to resources (Stephan 2010). The theories of human capital formation predict an inverse U-shape relationship between age and research productivity. Although gender has been found to affect research productivity, its impact seems to have decreased more recently (Xie and Shauman 2003).

The empirical literature explains research productivity, either at the individual or at the aggregated level, building on the specificities of the scientific reward system and on individual and institutional characteristics. In line with the human capital theories, Levin and Stephan (1991) and Rauber and Ursprung (2008) found positive age and cohort effects, and Maske, Durden and Gaynor (2003) significant gender differences. Collaboration also pays as demonstrated by a recent study of Bosquet and Combes (2013). Elhorst and Zigova (2014) showed that neighbouring economics departments compete in producing research output by identifying a robust negative spatial lag coefficient on average department productivity. Other studies found positive scale effects (e.g. Bonacorsi and Daraio 2005) and positive spillover effects stemming from good university location (Kim, Morse, and Zingales 2009).

In our empirical model we include career age, gender, level of collaboration, and type of academic position as possible productivity determinants at the individual level. Career age is measured by the number of years since PhD graduation. As the impact of age may be non-linear, we include both log of career age and log of career age squared. Gender effects are captured by a female dummy, while dummies for post-doc and junior professors control for productivity differences relative to full professors. Collaboration activity is measured by the share of externally coauthored papers to all papers, where an external coauthor is somebody from outside the affiliated university. The institutional variables are department size and publishing "culture" of the department. Like career age, department size enters the model as log and log squared to allow for potential and non-linear scale effects. The share of department members who did not publish any articles in a journal with non-zero quality weight over the relevant decade, represents the publication "culture" of the department. Following other studies focusing on German speaking countries (Fabel, Hein and Hofmeister 2008; Elhorst and Zigova 2014), we use country dummies for Swiss and Austrian departments to compare their productivity with their German counterpart.

Alternatively, we may hold out Swiss and Austrian departments and use these samples for post-fitting evaluation. However, to identify the model with heteroskedastic shocks, we require variation in department size. Since we have a relatively small number of universities (83) in the sample, taking a truly random sample of departments would risk weakening identification. Table 1 shows the size distribution of the departments in the sample. Another problem is that the size of departments will turn out to be both a significant determinant of research productivity and of the extent of heteroskedasticity. By holding out observations, these findings may get lost due to insufficient variation.

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We therefore propose to conduct the analysis with a pooled sample and suitable dummy variables to capture the sample split, followed by statistical tests on the significance of these dummies. The dummies for Swiss and Austrian departments will be used for this purpose.

New strands of empirical literature focus on measuring peer effects in academia using a natural experiment setting. Azoulay, Zivin and Wang (2010) measure productivity losses of collaborators of star scientists after an unexpected death. They estimate an up to 8% decrease in research productivity of American life scientists. On the contrary, Waldinger (2011), finds no evidence of peer effects applying in historical 1925-1938 productivity data of German scientists, who were colleagues of expelled Jewish faculty. One of the explanations Waldinger suggests is that scientists were much more specialized in the past, hence a loss of a peer might not affect individual productivities. A recent study by Borjas and Doran (2014) finds productivity losses of Soviet mathematicians exposed to vast emigration in the 1990s of their colleagues to the United States or to western Europe. Whereas the emigration of average collaborators appeared to have no effect on the research output of a mathematician, the emigration of just 10% of high-quality coauthors implied roughly a 8% percent decline. Our study adds another piece to the so far rather mixed evidence on peer effects in academia using the GNS model applied to non-experimental data.

4.2 GNS and modelling research productivity interactions

The concept of cumulative advantage in science (Carayol 2008) leads to weaker overall significance of models explaining research productivity, because observed individual and institutional variables cannot fully explain why research productivity among scientists is so skewed (Stephan 2010). The terms $\mathbf{WY}$, $\mathbf{WX}$ and/or $\mathbf{WU}$ in the GNS model, or in models nested within it, can add more explanatory power because they bear additional information on colleagues' average productivity, the determinants of their productivity, and common unobserved characteristics. In our setting, $\mathbf{X}$ consists of variables that vary at the individual and at the university level. Since the group interaction matrix $\mathbf{W}$ is block diagonal and the institutional variables do not vary over the department members working at the same university, pre-multiplying the institutional variables with the group interaction matrix would lead to an identical set of variables. For this reason we multiply $\mathbf{W}$ only with individual level variables. The condition that the matrix $\mathbf{X}^{*'}\mathbf{B}'\mathbf{\Omega}^{-1}\mathbf{B}\mathbf{X}^{*}$ should have full rank will also not be satisfied if group fixed effects are added, i.e., one

dummy for every group of researchers working at the same university. Due to perfect multicollinearity such fixed effects would absorb the effects of the institutional variables. This means that institutional variables need to be fully removed from the regression equation if group fixed effects are added.

Applying Elhorst's (2010) terminology to our setting, a significant endogenous effect (ρ_0) would mean that the productivity of an individual researcher depends on the productivity of department colleagues. Significant exogenous effects (γ_0) signal that somebody's productivity is influenced by observed characteristics of these colleagues, while correlated effects (λ_0) signal that individual productivity varies with unobserved characteristics common to all colleagues from one department. By estimating these parameters we could conclude on the existence, type, and extent of these localized peer effects. But as Waldinger (2011) points out, sorting of individuals complicates the estimation of peer effects, as highly productive scientists often choose to co-locate. Sorting may therefore introduce a positive correlation of scientists' productivities within universities not caused by pure peer effects. Since the spatial parameters ρ_0, λ_0 and γ_0 may be contaminated by sorting, because individuals "settle" in equilibrium at the best achievable university given their observed output, we need to be careful in interpreting the interaction parameters. By considering direct and indirect (spillover) effects (Section 2.3), especially regarding the publishing culture of a department, and different model specifications nested within GNS, we will nonetheless be able to draw conclusions regarding the kind of peer effects that drive research productivity within departments, as well as whether sorting matters. The overall effect of the publishing culture potentially consists of a direct effect and a spillover effect. The direct effect of this variable to research productivity reflects sorting; staff members self-select into departments with peers of similar quality and departments appoint new staff of similar productivity. The spillover effect of this variable measures the extent to which individual productivity is affected by that of its peers, including the impact of newly appointed colleagues. Since models in which $\rho_0 \neq 0$ cover this spillover effect and models with $\rho_0 = 0$ do not (see eq. 15), and these models can be tested against each other, we can draw conclusions regarding the existence of this peer effect in addition to sorting.

5 Estimation results

Table 2 reports our estimation results. We consider eight different model specifications, from the simplest OLS to the most complex GNS specification. The GNS model includes

all three types of interaction effects, while the other models nested within it lack one or more of these effects which explains the empty entries in Table 2. Figure 1 shows the restrictions (next to the arrows) that need to be imposed on the parameters of the GNS model to obtain these simpler models. This figure is taken from Halleck Vega and Elhorst (2015) and adjusted to the mathematical notation used in this paper. In addition, the SLX and the SEM models have been switched.

The weight matrix is specified as a block-diagonal matrix given by $\mathbf{W}_r = \frac{1}{n_r-1}[\mathbf{1}_{n_r}\mathbf{1}'_{n_r} - \mathbf{I}_{n_r}]$, where n_r is the number of researchers in the rth department ($r = 1, ..., 83$). In other words, we are using a row-normalized spatial weights matrix whose elements have a value of one if researchers are in the same department, and zero otherwise.

5.1   Model with group fixed effects

We first focus on group fixed effects. According to Lee, Liu and Lin (2010), the GFE-GNS model can be estimated using two log-likelihood functions defined in (4.1) or (4.2) of their paper. The first is based on transformed variables and the transformed spatial weights matrix $\mathbf{W}^*$. Since all eigenvalues of the transformed $\mathbf{W}^*$ are $\frac{-1}{n_r-1}$ for $r = 1, \ldots, \bar{r}$ (see Section 2.2), the upper bound of the interval on which the spatial autoregressive or spatial autocorrelation coefficients are defined is $1/(\frac{-1}{n_{max}-1})$, where $n_{max} = \max(n_r) = 160$ is the the size of the largest group in the sample (see Table 1). Since this upper bound is clearly greater than one, $1/|-1/(160-1)|$, we obtained parameter estimates exceeding 1 for the SAR, SEM, SDM, and SDEM model specifications; the largest estimate appeared to be 9.127.

The second log-likelihood is based on the original observations, adjusted for the reduction of the number of degrees of freedom. This approach keeps the upper bound of the interval on which the spatial autoregressive or spatial autocorrelation coefficients are defined at 1. Unfortunately, this helped only partly, because in this case we obtained unrealistic parameter estimates close to 1. For example, for the GNS model we estimated $\rho_0 = 0.910$ with t-value 0.59 and $\lambda_0 = 0.955$ with t-value 1.25. The explanation for these unrealistic findings is the presence of near multicollinearity between the $\mathbf{X}$ variables and their spatially lagged values, $\mathbf{WX}$, caused by the inclusion of group fixed effects. To further investigate this, we calculated the correlation coefficient for the six individual-specific variables (recall that the institutional variables are absorbed by the group fixed effects), which ranged from 0.9866 for the square of the career age variable up to 0.9961 for the dummy of junior professors. We mathematically predicted these high correlation

coefficients in (12). It should be stressed that this result hinges strongly on the group interaction matrix. If a different spatial weights matrix would be adopted, these group fixed effects may retrieve their significance again.

One may also leave the interaction effects aside and just control for the group fixed effects instead. The coefficient estimates of this model are comparable to those of the OLS model reported in the first column of Table 2: 0.002 for log career age, -0.013 for $\log^2$ career age, -0.083 for post-docs, -0.055 for junior professors, -0.028 for females, and 0.044 for collaboration. We will see shortly that these coefficient estimates are close to the direct effects derived from a broad range of spatial econometric models. One disadvantage of this model however is that it cannot provide information about potential spillover effects. Another drawback is that it does not provide information about the institutional variables, since they are absorbed by the group fixed effects.

In view of these outcomes, we endorse and follow Corrado and Fingleton's (2012) recommendation that it is better to retain the institutional variables than to introduce dummy variables that combine their effects with those of any omitted variables. Therefore Table 2 contains estimates of the eight models without group fixed effects.

5.2   Heteroskedasticity and model reduction

The second round of testing concerns heteroskedasticity and model reduction. In interpreting the evidence in Table 2, we consider the various likelihood ratios that are constructed as approximately Chi-square distributed with the usual degrees of freedom under the relevant null hypothesis.[8] We specified group heteroskedasticity as $\sigma_r^2 = \alpha_1 + \alpha_2 n_r$, where n_r is the size of the economics department measured by the number of people. The test for reduction to homoskedasticity thus means testing the hypothesis that $\alpha_2 = 0$, and therefore has one degree of freedom. The most general model, the HGNS, reduces to the GNS, under homoskedasticity. The likelihood ratio (LR) test statistic is equal to $2(2367.3 - 2359.0) = 16.6$ which is highly significant if treated as χ_1^2 under the null. This keeps the HGNS as the maintained model.

Next, we test for the HGNS model reductions to (i) the heteroskedastic SDM ($\lambda_0 = 0$) (1 d.f.) for which $LR = 2(2367.3 - 2367.3) = 0$ to within rounding error, or to (ii) the heteroskedastic SDEM ($\rho_0 = 0$) (1 d.f.) for which $LR = 2(2367.3 - 2367.0) = 0.6$, or to

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[8]The quality of this approximation obviously deserves some attention, but as will be apparent from the details, the conclusions would not be likely to change much if a more accurate reference distribution was available.

(iii) the heteroskedastic SAC ($\gamma_{\mathbf{0}} = \mathbf{0}$) (6 d.f.) for which $LR = 2(2367.3 - 2361.4) = 11.8$. Neither model reduction (i) or (ii) is rejected, while (iii) is rejected at 10% significance level.

Further simplification of the heteroskedastic SDM to the homoskedastic SDM is rejected by the likelihood ratio of $LR = 2(2367.3 - 2358.8) = 17.0$ (1 d.f.). Similarly, the reduction of the heteroskedastic SDM to the heteroskedastic SLX ($\rho_0 = \lambda_0 = 0$) (2 d.f.) gives $LR = 2(2367.3 - 2353.7) = 27.2$ and is clearly rejected. Reduction of the heteroskedastic SDEM to the homoskedastic SDEM is equally rejected by the likelihood ratio of $LR = 2(2367.0 - 2358.5) = 17.0$ (1 d.f.). Finally, the reduction of the heteroskedastic SDEM to the heteroskedastic SLX is also rejected. No further model reductions need to be tested, because already the simpler models nested by either the SDM or SDEM are rejected by the data. This strongly suggests that either the heteroskedastic SDM or SDEM could serve as the maintained model. Given that heteroskedastic specifications outperform the homoskedastic ones for the three non-rejected models, Table 2 contains estimates of the eight models with group heteroskedastic disturbances.

5.3   Direct and indirect effects

We now turn our attention to an interpretation and comparison of the results for the heteroskedastic GNS, SDM and SDEM models.[9] We consider the estimates of the direct and indirect (spillover) effects of the different explanatory variables to see whether they can be used as an alternative means to select the best model from the three non-rejected models. Table 3 reports the estimates of the direct effects of the explanatory variables of the different models. A direct effect represents the impact of a change in one $\mathbf{X}$ variable of the average researcher on the productivity of the average researcher, measured by the mean of $\mathrm{DE}_{\beta_0}(r) + \mathrm{DE}_{\gamma_0}(r)$ in equations (14) and (17) over all r.

The general pattern that emerges from Table 3 is the following. The differences between the direct effects and the coefficient estimates reported in Table 2 are generally very small. In the rejected OLS, SEM, and SLX and non-rejected SDEM models they are exactly the same by definition; in the rejected SAR, and SAC models and the non-rejected SDM and

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[9]As an alternative to the LR tests for homoskedasticity one may also estimate the homoskedastic model and then carry out the Breusch-Pagan test for heteroskedasticity. The outcomes of this LM-test range from 3.46 in the SAR model to 4.26 in the SEM model with one degree of freedom. The evidence in favour of heteroskedasticity from this perspective is slightly weaker than from the perspective of the more powerful LR-test.

GNS models they may be different due to the feedback from the endogenous interaction effects ($\rho\mathbf{WY}$). Empirically, however, these feedback effects appear to be very small.

In the three non-rejected models, the differences between the direct effects are in most cases also very small. But, the GNS model clearly suffers from inefficiency as all of its estimates are insignificant, even if the size of the direct effect is in most cases of the same magnitude as in the SDM and the SDEM. For instance the coefficients of the variable 'No top publishers' (varies at the university level) are similar for the GNS and SDM models, but in the GNS model the effect is insignificant. Similarly, the coefficient of the dummy for 'Junior professor' (varies at the individual level) is around -0.054 in all three models, but it is only significant in the SDM and SDEM. Another notable exception is the 'log^2(career age)' which has a significant and sizeable direct effect estimate of less than -1.0 in the SDM and SDEM, but a negligible and insignificant direct effect estimate of about -0.01 in the GNS. From these inspections it is clear that the results for the SDM and SDEM models are more consistent with each other rather than with the GNS model that nests them.

The importance of basing inferences on the estimates from the non-rejected GNS, SDM, SDEM models, can be clearly seen in the case of the 'Switzerland' and the 'log size' effects. An analyst using the results from OLS, SAR, SEM or SLX, i.e. models that cover at most one type of interaction effects, would conclude that researchers in Switzerland are more productive than in Austria and Germany, and so the researchers employed by larger departments, while analysts adopting the SDM, SDEM, or GNS model would not. Since only the SDM, SDEM and the GNS models are not rejected by the data, the former group of analysts in this case would be basing their calculations, and hence their contrary conclusions, on a rejected model. Furthermore, they might erroneously conclude that holding out Swiss and Austrian departments or departments of certain size classes would lead to different outcomes when carrying out post-fitting evaluations.

The levels of the t-values reported for the direct effects of variables that vary at the individual level (Table 3) are almost the same in all models, except for the SAC and the GNS models. In the SAC model it halves in most cases, while in the GNS model it always drops (in absolute value) below 1. The explanations for this is that the significance level of the endogenous peer effect coefficient (ρ_0) of the $\mathbf{WY}$ variable in the SAC and the GNS models falls considerably, presumably because this variable competes in these two models for significance with the interaction coefficient (λ_0) of the disturbance $\mathbf{WU}$. Additionally, for the GNS model we observe that also the t-values of the explanatory variables that vary at the university level (see Table 2) decrease so much that all these variables become

insignificant and therefore also the respective direct effects reported in Table 3. To some extent this also applies to the spatially lagged values of the explanatory variables in the GNS model.

Table 4 reports the spillover effects of the explanatory variables of the different models. A spillover effect represents the impact of a change in one $\mathbf{X}$ variable of the average researcher on the productivity of other researchers working at the same university. It is measured by the mean of $IE_{\beta_0}(r) + IE_{\gamma_0}(r)$ in equations (15) and (18) over all r. In contrast to the direct effects, the differences between the estimated spillover effects in the different models are very large. Nevertheless, we can observe some general patterns. The rejected OLS, SAR, SEM and SAC models produce no or contradictory spillover effects compared to the SDM, SDEM and GNS models. For example, whereas the spillover effect of post-docs in the SDM and SDEM models is positive and significant, it is zero by construction in the OLS and SEM models, negative in the SAC model, and negative and "significant" in the SAR model. The negative but insignificant effect in the SAC model can be explained by the fact that this model closely resembles the SEM model; as in the SAC the autoregressive coefficient of $\mathbf{WY}$ is so small that spillover effects cannot occur in this model. The negative and significant effect in the SAR model can be explained by the fact that in this model the ratio between the spillover effect and the direct effect is the same for each explanatory variable (Elhorst 2010). Consequently, this model is too rigid to model spillover effects adequately, and is, of course, rejected by the data.

The spillover effects identified by the rejected SLX and the non-rejected SDM, SDEM and GNS models are of the same order of magnitude, at least for the variables that vary at the individual level. By construction there are no spillover effects for the variables that vary at the university level for the SLX and SDEM models. The t-values in the SLX model are however clearly too high, because this model ignores interaction effects either among the dependent variable or the error terms. The t-values of the spillover effects of the SDM and the SDEM models are of the same order of magnitude, while they are insignificant in the GNS model. For example, according to the SDM, SDEM and GNS models, the spillover effect of post-docs ranges from 0.086 to 0.089, and is therefore rather stable, whereas the t-values in the first two models are 2.32 and 2.13, respectively, and in the last model only 0.02. As recently pointed out by Gibbons and Overman (2012), the explanation for this finding is that interaction effects among the dependent variable and interaction effects among the error terms are only weakly identified. Considering them both, as in the GNS model, highlights this problem; it leads to a model that is overparameterised, which reduces the significance levels of all variables. This finding

is worrying since the interpretation of the two types of interaction effects is completely different. In our case, a model with endogenous interaction effects posits that the research productivity of a researcher depends on the research productivity of other researchers working at the same university, and vice versa. By contrast, a model with interaction effects among the error terms assumes that the research productivity of a researcher depends on unobserved characteristics that affect all researchers working at the same university.

Although the SDM and SDEM specifications produce spillover effects that are of the same order of magnitude and significance for the variables measured at the individual level, the results reported in Tables 3 and 4 indicate that this does not hold for the variable 'No top publishers' that varies only across universities and measures the publication culture of a university. According to the SDEM specification, a unit change in the proportion of colleagues who do not publish in top journals appears to have a negative total/direct effect on productivity of 0.178; the SDM specification produces an almost similar negative total effect of 0.181, but according to this model it can be split up into a negative direct effect of 0.127 on individual researchers (Table 3) and a negative spillover effect of 0.054 on other researchers within the same university (Table 4).

Although the total effects of the explanatory variables, the sum of the direct effects and the corresponding spillover effects, and their significance levels have been calculated, they are not reported to save space. Generally, we find that if the direct effect is insignificant, the total effect is also insignificant. The total effect of a particular variable is also insignicant if its direct effect and its spillover effect have opposite signs. Finally, if the direct effect is significant and the spillover effect has the same sign, then the total effect is also (weakly) significant. This holds for 'no top publishers' and 'collaboration', indicating that researchers working on papers with external coauthors and in departments with a strong publishing culture tend to be more productive.

5.4   Choice between the SDM and SDEM

Ideally the GNS model should serve as a means of selection between the SDM and SDEM models, but given the demonstrated weak identification of this model a Bayesian perspective on whether either the SDM or the SDEM specification generated the data is more appropriate. We apply a novel Bayesian approach here taken from LeSage (2014). By addressing the marginal likelihood of both specifications, and thereby integrating out all parameters of the model, we calculate the Bayesian posterior model probabilities of the

SDM and SDEM specifications, conditional on the sample data. With the two tested models, we have $p(\text{SDM}|\mathbf{Y}, \mathbf{X}^*, \mathbf{W}) + p(\text{SDEM}|\mathbf{Y}, \mathbf{X}^*, \mathbf{W}) = 1$. If the probability of one model is greater than that of the other, we conclude that it describes the data better, because the comparison is based on the same set of explanatory variables, that is, both model specifications include $\mathbf{X}$ and spatially lagged $\mathbf{X}$ (denoted by $\mathbf{WX}$) variables, and the comparison is independent of any specific parameter values as they have been integrated out.

The main strength of this Bayesian approach is that it compares the performance of one model against another model, in this case SDM against SDEM, on their entire parameter space. The popular likelihood ratio, Wald and/or Lagrange multiplier statistics only compare the performance of one model against another model based on specific parameter estimates within the parameter space. Inferences drawn on the log marginal likelihood function values for the SDM and SDEM model are further justified because they have the same set of explanatory variables, $\mathbf{X}$ and $\mathbf{WX}$, and are based on the same uniform prior for ρ_0 and λ_0. This prior takes the form $p(\rho_0) = p(\lambda_0) = 1/D$, where $D = 1/\omega_{\max} - 1/\omega_{\min}$ and $\omega_{\max}$ and $\omega_{\min}$ represent respectively the largest and the smallest (negative) eigenvalue of the spatial weights matrix $\mathbf{W}$. This prior requires no subjective information on the part of the practitioner as it relies on the parameter space $(1/\omega_{\min}, 1/\omega_{\max})$ on which ρ_0 and λ_0 are defined, where $\omega_{\max} = 1$ if $\mathbf{W}$ is row-normalized. Full details regarding the choice of model can be found in LeSage (2014).

The Bayesian posterior model probabilities based on this approach are found to be in the proportion of 0.0124 for the SDM specification and 0.9876 for the SDEM specification, indicating that it is almost 80 times more likely that the interaction effect that has been found in addition to exogenous interaction characteristics ($\mathbf{WX}$) is due to unobserved characteristics common to all colleagues within a department ($\mathbf{WU}$) rather than that peers affect the productivity of colleagues ($\mathbf{WY}$). Consequently, we may conclude that only one variable produces significant spillover effects within a department is the presence of post-docs (see Table 4). Post-docs appear to publish less than junior professors, who in turn publish less than senior staff members, but they do have a positive effect on their environment; a post-doc within a department has a positive spillover effect of 0.086 on the research productivity of his or her colleagues. Since the SDEM specification is more likely than the SDM specification, a unit change in the proportion of colleagues who do not publish in top journals may be said not to produce a spillover effect, rejecting peer effects and reflecting the importance of sorting. Suppose that due to department policy changes more researchers become active in publishing, as a result of which the proportion

of colleagues who do not publish within the department decreases. This would cause a shock to the equilibrium situation and lead to a reshuffling of researchers. Not only will more productive researchers join the department, due to the absence of peer effects they will probably also replace inactive or unproductive colleagues since the latter are not able to benefit from this productivity impulse. This prediction follows from rejecting peer effects (SDM specification) and is in line with recent studies of Waldinger (2011) and Borjas and Doran (2014) using a natural experiment setting, who also found no or only small localized peer effects.

6   Conclusions

This paper is among the first to study the theoretical model of group interactions suggested by Lee, Liu and Lin (2010) in an empirical setting, throwing more light on its feasibility, empirical relevance, and its empirical implications. Based on this study, the current unpopularity of the GNS model with a full set of interaction effects among the dependent variable, the exogenous variables, and among the disturbances, can be explained by following two reasons, of which especially the second has not yet been empirically documented in the literature.

The first reason is that general conditions under which the parameters of the GNS model are identified have only recently been given, by Lee, Liu and Lin (2010) for a specific form of spatial weights, namely a group interaction matrix. Unfortunately, this matrix is not very popular in applied spatial econometric research. The second reason is that the GNS model can be overparameterised which leads to weak identification of the interaction effects among the dependent variable and among the error terms. Considering them both, as in the GNS model, has the effect that the significance levels of all variables go down, and hence the GNS model provides no additional information over the nested SDM and SDEM specifications. This implies that the potential advantage of the GNS model that either of the SDM or SDEM could be rejected against does not hold from an empirical perspective.

This paper also goes a step further than the general nesting spatial (GNS) model with all types of interaction effects set out in Lee, Liu and Lin (2010). Firstly, we show that spatial econometric models with limited numbers of spatial interaction effects lead to incorrect inferences. This justifies the path to more general models in empirical modelling. The spillover effects produced by the SAR, SEM, SLX and SAC models, often the main

focus of the analysis, are demonstrably false. A much better performance is obtained when adopting the SDM or the SDEM model.

Secondly, whereas Lee, Liu and Lin (2010) advocate the extension of the GNS model with group fixed effects, we provide evidence, both mathematically and empirically, that this extension has hardly any empirical relevance due to near multicollinearity. By contrast, we find a strong evidence in favour of heteroskedasticity; the heteroskedastic models outperform their homoskedastic counterparts, signalling that spatial econometricians should devote more attention to accounting for heteroskedasticity. Although Anselin (1988) advocated the incorporation of heteroskedastic disturbances in spatial econometric models over twenty-five years ago, only a few empirical studies have appeared since then that followed his call. By making our routines downloadable for free, we hope to stimulate more such studies.

Inability to decide between the SDM and SDEM specifications based on the GNS estimates, implies that more information is needed to discriminate between the two types of interaction effects described by these models. By taking a Bayesian approach (LeSage 2014) we were able to show that the SDEM specification is more appropriate. This specification predicts a small positive and significant spillover effect from the presence of post-docs but no spillover effects from non-publishing faculty. These results are in line with experimental studies on peer effects in academia, finding no effects (Waldinger 2011) or no effects for average collaborators (Borjas and Doran 2014).

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Table 1: Size distribution across the universities

Size	Frequency
10-19	22
20-29	32
30-39	12
40-49	4
50-59	6
60-99	5
100	1
160	1

Table 2: Explaining log research productivity using different model specifications

Determinants	OLS	SAR	SEM	SLX	SDM	SDEM	SAC	GNS
intercept	0.006	0.011	-0.005	0.344*	0.236	0.326	-0.007	0.262
	(0.08)	(0.16)	(-0.05)	(1.81)	(1.24)	(1.16)	(0.07)	(0.64)
Austria	0.004	0.009	0.004	-0.003	-0.002	-0.002	0.004	-0.002
	(0.50)	(1.28)	(0.40)	(-0.25)	(-0.16)	(-0.13)	(0.32)	(-0.13)
Switzerland	0.018**	0.013*	0.018*	0.014*	0.010	0.015	0.018	0.011
	(2.43)	(1.74)	(1.68)	(1.75)	(1.25)	(1.34)	(1.45)	(0.64)
log size	0.087**	0.063*	0.089*	0.060*	0.043	0.063	0.092	0.049
	(2.70)	(1.96)	(1.90)	(1.79)	(1.28)	(1.37)	(1.61)	(0.64)
$\log^2$ size	-0.009**	-0.006	-0.009	-0.006	-0.005	-0.007	-0.009	-0.005
	(-1.99)	(-1.43)	(-1.44)	(-1.40)	(-1.01)	(-1.10)	(-1.26)	(-0.60)
no top publishers	-0.183**	-0.110**	-0.184**	-0.183**	-0.126**	-0.178**	-0.188*	-0.140
	(-11.26)	(-5.58)	(-7.11)	(-8.61)	(-5.43)	(-5.60)	(-2.29)	(-0.73)
log career age	0.001	0.005	0.008	0.004	0.007	0.004	0.008	0.006
	(0.03)	(0.21)	(0.33)	(0.15)	(0.27)	(0.15)	(0.34)	(0.22)
$\log^2$ career age	-0.010*	-0.011*	-0.011**	-0.010*	-0.011**	-0.010*	-0.011**	-0.011*
	(-2.06)	(-2.26)	(-2.39)	(-2.19)	(-2.34)	(-2.17)	(-2.42)	(-2.08)
post-doc	-0.075**	-0.077**	-0.078**	-0.077**	-0.078**	-0.077**	-0.079**	-0.078**
	(-12.66)	(-13.02)	(-13.17)	(-13.03)	(-13.28)	(-12.98)	(-13.26)	(-11.56)
junior professor	-0.051**	-0.052**	-0.053**	-0.053**	-0.054**	-0.053**	-0.053**	-0.054**
	(-7.57)	(-7.86)	(-8.06)	(-7.97)	(-8.10)	(-8.09)	(-7.98)	(-7.87)
female	-0.027**	-0.027**	-0.027**	-0.027**	-0.027**	-0.027**	-0.027**	-0.027**
	(-5.04)	(-5.08)	(-5.16)	(-5.06)	(-5.11)	(-5.09)	(-5.14)	(-5.09)
collaboration	0.045**	0.044**	0.043**	0.043**	0.043**	0.043**	0.043**	0.043**
	(8.38)	(8.30)	(8.02)	(8.03)	(8.04)	(7.95)	(8.14)	(7.84)
$W \cdot Y$		0.323**			0.303**		-0.017	0.226
		(6.55)			(5.65)		(-0.05)	(0.22)
$W \cdot$ log career age				-0.278*	-0.199	-0.276		-0.222
				(-2.05)	(-1.46)	(-1.37)		(-0.70)
$W \cdot \log^2$ career age				0.057*	0.044*	0.057		0.048
				(2.25)	(1.73)	(1.52)		(0.86)
$W \cdot$ post-doc				0.092**	0.087**	0.086*		0.087**
				(3.37)	(3.20)	(2.13)		(2.79)
$W \cdot$ junior prof.				0.037	0.042	0.036		0.040
				(1.14)	(1.30)	(0.74)		(0.96)
$W \cdot$ female				0.008	0.014	0.009		0.013
				(0.28)	(0.51)	(0.22)		(0.36)
$W \cdot$ collaboration				0.026	0.008	0.035		0.015
				(0.96)	(0.30)	(0.88)		(0.18)
$W \cdot U$			0.350**			0.300**	0.363	0.102
			(6.99)			(5.57)	(1.63)	(0.08)
intercept *(Het.)*	0.650**	0.646**	0.644**	0.646**	0.641**	0.640**	0.656**	0.661**
	(14.65)	(14.81)	(14.95)	(14.71)	(14.79)	(14.90)	(14.71)	(11.67)
size*(Het.)*/100	0.698**	0.666**	0.663**	0.684**	0.664**	0.666**	0.634**	0.620**
	(6.86)	(6.70)	(9.19)	(6.81)	(6.72)	(9.26)	(6.47)	(8.41)
Log Lik. *(Het.)*	2340.8	2357.4	2361.3	2353.7	2367.3	2367.0	2361.4	2367.3
Log Lik. *(Hom.)*	2331.9	2349.0	2352.8	2345 .1	2358.8	2358.5	2352.8	2359.0
p-val LR-test Het/Hom	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002

**significant at 5%, *significant at 10%, t-values in parentheses

Table 3: Direct effects on research productivity of individual researchers

Determinants	OLS	SAR	SEM	SLX	SDM	SDEM	SAC	GNS
Austria	0.004	0.010	0.004	-0.003	-0.003	-0.002	0.004	-0.002
	(0.50)	(1.28)	(0.40)	(-0.25)	(-0.20)	(-0.13)	(0.33)	(-0.04)
Switzerland	0.018**	0.012*	0.018*	0.014*	0.010	0.015	0.018	0.011
	(2.43)	(1.74)	(1.68)	(1.75)	(1.23)	(1.34)	(1.41)	(0.17)
log size	0.087**	0.081*	0.089*	0.060*	0.059	0.063	0.092*	0.049
	(2.70)	(1.94)	(1.90)	(1.79)	(1.32)	(1.37)	(1.72)	(0.62)
$\log^2$ size	-0.009*	-0.009	-0.009	-0.006	-0.007	-0.007	-0.009	-0.005
	(-1.99)	(-1.42)	(-1.44)	(-1.40)	(-1.04)	(-1.10)	(-1.35)	(-0.10)
no top publishers	-0.183**	-0.113**	-0.184**	-0.183**	-0.127**	-0.178**	-0.188*	-0.140
	(-11.26)	(-5.41)	(-7.11)	(-8.61)	(-5.31)	(-5.60)	(-2.26)	(-0.78)
log career age	0.001	0.006	0.008	0.004	0.003	0.004	0.008	0.003
	(0.03)	(0.16)	(0.33)	(0.15)	(0.13)	(0.15)	(0.32)	(0.09)
$\log^2$ career age	-0.010*	-0.011*	-0.011**	-0.010*	-0.010*	-0.010*	-0.011*	-0.010
	(-2.06)	(-2.17)	(-2.39)	(-2.19)	(-2.12)	(-2.17)	(-2.24)	(-0.87)
post-doc	-0.075**	-0.080**	-0.078**	-0.077**	-0.079**	-0.077**	-0.079**	-0.077
	(-12.66)	(-12.70)	(-13.17)	(-13.03)	(-13.48)	(-12.98)	(-5.12)	(-0.70)
junior professor	-0.051**	-0.053**	-0.053**	-0.053**	-0.054**	-0.053**	-0.053**	-0.054
	(-7.57)	(-7.70)	(-8.06)	(-7.97)	(-8.05)	(-8.09)	(-4.38)	(-0.70)
female	-0.027**	-0.028**	-0.027**	-0.027**	-0.028**	-0.027**	-0.027**	-0.027
	(-5.04)	(-5.10)	(-5.16)	(-5.06)	(-5.00)	(-5.09)	(-3.80)	(-0.34)
collaboration	0.045**	0.046**	0.043**	0.043**	0.045**	0.043**	0.043**	0.043
	(8.38)	(8.42)	(8.02)	(8.03)	(8.41)	(7.95)	(4.07)	(0.60)

**significant at 5%, *significant at 10%, t-values in parentheses

Table 4: Spillover effects of individual research productivity on research colleagues at the same university

Determinants	OLS	SAR	SEM	SLX	SDM	SDEM	SAC	GNS
Austria		0.005			-0.001		-0.008	-0.001
		(1.21)			(-0.19)		(-0.06)	(-0.04)
Switzerland		0.006*			0.004		0.002	0.003
		(1.65)			(1.14)		(0.04)	(0.03)
log size		0.038*			0.025		0.002	0.014
		(1.77)			(1.23)		(0.02)	(0.02)
$\log^2$ size		-0.004			-0.004		-0.000	-0.002
		(-1.33)			(-0.98)		(-0.04)	(-0.01)
no top publishers		-0.053**			-0.054**		0.011	-0.040
		(-5.08)			(-3.98)		(0.14)	(-0.04)
log career age		0.003		-0.278*	-0.344	-0.276	0.003	-0.282
		(0.15)		(-2.05)	(-1.30)	(-1.37)	(0.04)	(-0.09)
$\log^2$ career age		-0.005*		0.057*	0.069	0.057	-0.003	0.058
		(-1.95)		(2.25)	(1.45)	(1.52)	(-0.06)	(0.05)
post-doc		-0.037**		0.092**	0.086*	0.086*	-0.021	0.089
		(-4.20)		(3.37)	(2.32)	(2.13)	(-0.07)	(0.02)
junior professor		-0.025**		0.037	0.023	0.036	-0.015	0.035
		(-3.83)		(1.14)	(0.82)	(0.74)	(-0.06)	(0.01)
female		-0.013**		0.008	0.016	0.009	0.008	0.001
		(-3.45)		(0.28)	(0.22)	(0.22)	(-0.07)	(0.02)
collaboration		0.021**		0.026	0.032	0.035	0.013	0.032
		(4.00)		(0.96)	(0.76)	(0.88)	(0.06)	(0.03)

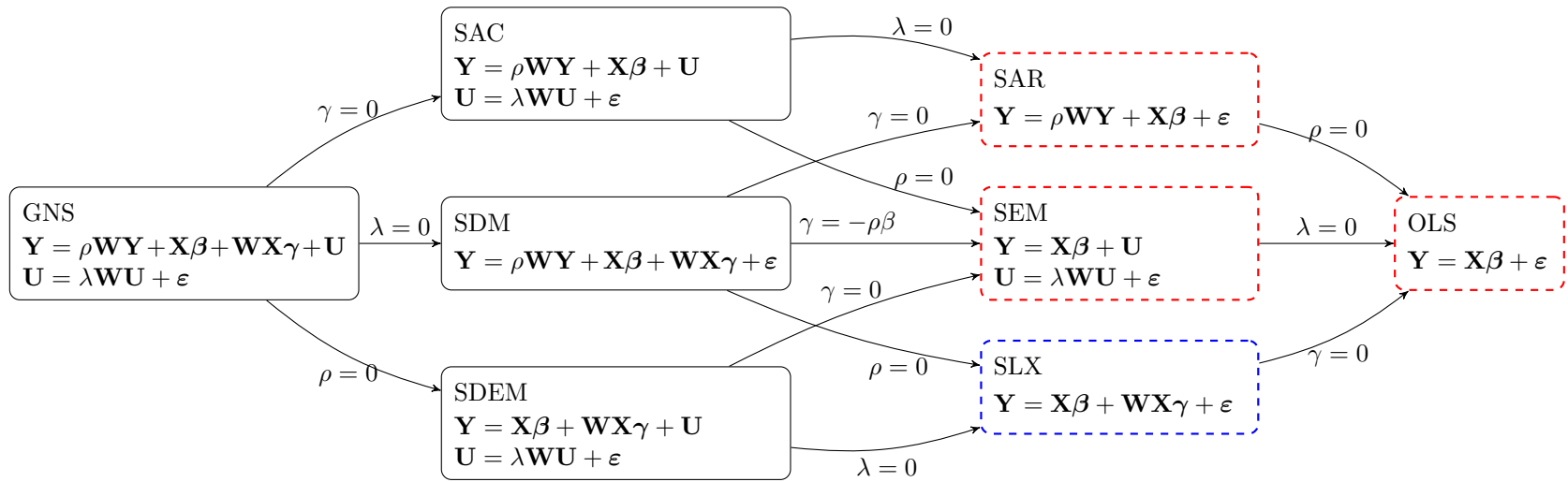**significant at 5%, *significant at 10%, t-values in parentheses

Figure 1: The relationships between different spatial dependence models for cross-sectional data. Subscripts 0 are left aside for simplicity. Adapted from Halleck Vega and Elhorst (2015)