The dynamic general nesting spatial econometric model for spatial panels with common factors: Further raising the bar

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The purpose of this presentation is to encourage more researchers to start using the general nesting spatial (GNS) econometric model for spatial panels with common factors (CF).

The dynamic general nesting spatial (GNS) econometric model for spatial panels with common factors (CF) reads as

$$Y_t = \boldsymbol{\tau Y_{t-1}} + \boldsymbol{\rho W Y_t} + \boldsymbol{\eta W Y_{t-1}} + X_t\beta + \boldsymbol{W X_t \theta} + \sum_r \boldsymbol{\Gamma_r^T f_{rt}} + u_t, \quad u_t = \boldsymbol{\lambda W u_t} + \varepsilon_t$$

Spatial lags are in red: 1+*K*+1=*K*+2 in total (*K* is number of *X* variables)

Dynamic effects are in green: 2 in total

Common factors are in blue: # parameters depend on type of CF, they are reported below.

In this presentation, the rationale behind each term that is part of the model, as well as potential objections or pitfalls of including certain terms from a statistical or economic-theoretical viewpoint, are explained. The purpose of this presentation is to encourage more researchers to start using this model, but in a responsible way.

$\boldsymbol{WY_t}$

A spatial lag in the dependent variable implies that y_{it} observed in cross-sectional unit *i* is explained by y_{jt} in other cross-sectional units *j*, $j \neq i$, and vice versa (two-way relationship).

The units *j* which are included depend on the specification of the spatial weight matrix *W*.

A linear regression model that contains a spatial lag in the dependent variable only ($\boldsymbol{WY_t}$) is known as a spatial autoregressive (SAR) model. The SAR model is one of the most widely used spatial econometric models to introduce new methods of estimation or of spatial statistics

Other models used for this purpose: SEM ($\boldsymbol{Wu_t}$) and SAC=SARAR ($\boldsymbol{WY_t}$, $\boldsymbol{Wu_t}$).

Estimation (SAR model)

Ord (1975): maximum likelihood (ML) estimator.

Anselin (1988), Kelejian and Prucha (1998, 1999): instrumental variables (IV) and generalized method-of-moments (GMM) estimators + regularity conditions W.

Lee (2004): quasi maximum likelihood (QML) estimator + regularity conditions W.

LeSage and Pace (2009, Ch.5): Bayesian Markow Chain Monte Carlo (MCMC) estimator (improvement of LeSage, 1997).

Bao and Ullah (2007): finite sample properties of ML estimator.

Ahrens and Bhattarchajee (2015): Lasso 2SLS estimator.

Kyriacou et al. (2017): Indirect Inference (II) estimator.

Smirnov (2021): closed-form consistent estimator based on ML.

Economic-theoretical underpinnings of $\boldsymbol{WY_t}$

-Anselin (2006): Conceptualization of strategic interaction or a spatial reaction function, $y_i = R(y_{\_i}, x_i)$, where $y_{\_i}$ reflects decisions by other agents. See for strategic interaction among local governments (Wildasin, 1988; Besley and Case, 1995; Brueckner, 2003, 2006; Allers and Elhorst, 2011).

-Pinkse et al. (2002) and LeSage et al. (2017): When one petrol station decreases its price, geographically nearby service stations need to follow in order not to lose market share.

-Hanson (2005): augmented market-potential function derived from Krugman's model of economic geography, reflecting the impact of scale economies and transport costs, explaining wages.

-Behrens et al. (2012): a quantity-based structural gravity equation system in which both trade flows and error terms are cross-sectionally correlated.

-Blonigen et al (2007): foreign direct investments (FDI).

-Xu and Lee (2019): SAR can be regarded as a model on the Nash equilibrium of a static complete information game with a linear-quadratic utility function.

Critique on including $\boldsymbol{WY_t}$

Moran's I test is unfocused and (robust) LM tests do not control for $\boldsymbol{WX_t}$ variables. Corrado and Fingleton (2012): $\boldsymbol{WY_t}$ variable picks up omitted $\boldsymbol{WX_t}$ variables or nonlinearities in the X variables.

Pinkse and Slade (2010): SAR model is fit onto every empirical problem. Entire spatial dependence structure is reduced to one single unknown coefficient.

McMillen (2012): SAR model (or SE model) is used as a quick fix for nearly any model related to space.

Elhorst (2010): $\left[\frac{\partial E(Y_t)}{\partial X_{1kt}} \dots \frac{\partial E(Y_t)}{\partial X_{Nkt}}\right] = ((1-\tau)I_N - (\rho+\eta)W)^{-1} I_N \beta_k$.

Ratio indirect and direct effect: the same for every explanatory variable in SAR model.

Halleck Vega and Elhorst (2015): $\boldsymbol{WY_t}$ causes global spillovers (indirect effects). Arbia and Fingleton (2008), Gibbons and Overman (2012), Corrado and Fingleton (2012), Partridge et al. (2012), Lacombe and LeSage (2015), and Elhorst et al. (2020): Difficult to form a reasonable argument for global spillover effects and thus $\boldsymbol{WY_t}$.

Dynamics: $\boldsymbol{Y_{t-1}}$ and $\boldsymbol{WY_{t-1}}$

Habit persistence. It takes time to change behavior.

Korniotis (2010): Internal and external habit persistence.

Anselin et al. (2008): time-space recursive spatial econometric model ($\boldsymbol{WY_t}$ not included). Suitable to explain spatial diffusion phenomena. Think of the rise and spread of the Covid-19 virus on a daily basis.

Goyal (2009, ch. 5): the social or spatial reaction function may take the form $y_{it} = R(y_{it-1}, y_{\_it-1}, x_i)$, $y_{\_it-1}$ reflects decisions by other agents in the previous period.

LeSage and Pace (2009, ch. 7): spatiotemporal (partial adjustment) model. High temporal dependence and low spatial dependence might nonetheless imply a long-run equilibrium with high spatial dependence.

Fogli and Veldkamp (2011): Information diffusion can change preferences (female labor force participation), but that people require time to gather information, creating a delay in the decision-making process, and hence spatial dependence takes time to manifest itself.

Restrictions on parameters

Coefficients of $\boldsymbol{Y_{t-1}}$, $\boldsymbol{WY_t}$ and $\boldsymbol{WY_{t-1}}$: $\tau + \rho + \eta < 1$.

If $\boldsymbol{WY_t}$ is not included: $\tau + \eta < 1$. Fogli and Veldkamp (2011): $0.916 + 0.570 > 1$.

Halleck Vega and Elhorst (2017): 0.845+0.019 for the total working population, 0.875+0.014 for the male, and 0.928+0.004 for the female working population.

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Parent and LeSage (2011, 2012): $\eta = -\tau\rho$.

Elhorst (2010): Impact of explanatory variables falls by the factor ρW for every higher-order neighbor, and by the factor τ for every next time period.

Lee and Yu (2015): Separable space-time filter.

Lee and Yu (2015): Limitations of $\eta = -\tau\rho$

Restriction is not necessary from an economic-theoretical viewpoint.

If $\tau = 0$ or $\rho = 0$, the spatiotemporal lag $\boldsymbol{WY_{t-1}}$ will automatically also have no effect since $\eta = 0$, which rules out diffusion and external habit persistence as in Korniotis (2010).

The omission of $\boldsymbol{WY_{t-1}}$ causes inaccuracy in forecasting when this variable is part of the true but unknown data generating process.

Rules out the possibility that ρ and η have the same sign, provided that τ is positive.

$\boldsymbol{WX_t}$

Halleck Vega and Elhorst (2015) and Elhorst and Halleck Vega (2017):

1. Since there are K $\boldsymbol{WX_t}$, and only one $\boldsymbol{WY_t}$ and only one $\boldsymbol{Wu_t}$, applied researchers better focus on $\boldsymbol{WX_t}$ variables first.
2. SAR ($\boldsymbol{WY_t}$), SE ($\boldsymbol{Wu_t}$) and SAC=SARAR ($\boldsymbol{WY_t}, \boldsymbol{Wu_t}$) models are of limited use in empirical research due to initial restrictions on the spillover effects they can potentially produce. Ratio between indirect and direct effect is zero (SEM) or the same for every explanatory variable (SAR, SAC).
3. *W* can easily be parameterized ($w_{ij} = 1/d^{\gamma}$).
4. Since $\boldsymbol{WX_t}$ variables do not cause severe additional econometric problems (regularity conditions), endogeneity of X_t and WX_t variables can be tested for using standard (non-spatial) IV tests.

Economic-theoretical underpinning of $\boldsymbol{WX_t}$ variables

LeSage and Pace (2009): Several motivations, though mostly statistical.

Ertur and Koch (2007): SD model of GDP per capita growth (initial income level, savings rate, population growth rate).

Yesilyurt and Elhorst (2017): SD model of military expenditures as a ratio of GDP.

Firmino Costa da Silva et al. (2017): dynamic SD and GNS model of a spatially augmented population growth model.

Heijnen and Elhorst (2018): SD diffusion model of waste disposal taxes across municipalities.

Xu and Lee (2019): game-theoretical model can be extended with $\boldsymbol{WX_t}$ variables.

$\boldsymbol{Wu_t}$

Determinants omitted from the model are spatially autocorrelated, or unobserved shocks follow a spatial pattern.

$\boldsymbol{Wu_t}$ affects efficiency but not the consistency of the parameter estimates.

Research option: This property is underused to test for misspecification problems.

Pace and LeSage (2008): Hausman test to compare OLS and SEM estimates.

Elhorst and Halleck Vega (2017): Three outcomes parameter estimates:

1. OLS ≈ SEM, λ not significant. Choose OLS.
2. OLS ≈ SEM, λ is significant. Choose SEM, no misspecification problems.
3. OLS ≠ SEM, λ is significant. Misspecification problems, respecify model.

Similar test can be carried out for static and dynamic SLX ($\boldsymbol{WX_t}$) and SDEM ($\boldsymbol{WX_t}$,$\boldsymbol{Wu_t}$), and SDM ($\boldsymbol{WY_t}$, $\boldsymbol{WX_t}$) and GNS ($\boldsymbol{WY_t}$, $\boldsymbol{WX_t}$, $\boldsymbol{Wu_t}$) models.

$\sum_r \boldsymbol{\Gamma_r^T f_{rt}}$

Option 1: Two factors $f_{1t} = (1, \ldots, 1)^T$ and $f_{2t} = (\xi_1, \ldots, \xi_T)^T$, with parameters of respectively $\Gamma_1^T = (v_1, \ldots, v_N)$ and $\Gamma_2^T = (1, \ldots, 1)$, gives a dynamic GNS model with cross-sectional and time-period specific effects. Number of CF parameters: N+T-1 (fixed effects), 2 (random effects).

v: vector of cross-sectional fixed or random effects

$\boldsymbol{\xi_t}$: time period fixed or random effects (t=1,…,T)

Baltagi (2005, pp. 66-68) and Lee and Yu (2012): Hausman specification test fixed vs. random.

However, fixed effects model is more appropriate from an econometric-theoretical viewpoint since the idea that a limited set of units or a limited set of time periods is sampled from a larger population must be rejected. Spatial econometricians tend to work with unbroken study areas (*W*) and consecutive time spans (dynamics).

Option 2: Keep the cross-sectional fixed effects, but replace the time dummies by cross-sectional averages (CSAs): $\bar{Y}_t = \frac{1}{N}\sum_{i=1}^{N} Y_{it}$, $\bar{Y}_{t-1} = \frac{1}{N}\sum_{i=1}^{N} Y_{it-1}$, and $\bar{X}_{kt} = \frac{1}{N}\sum_{i=1}^{N} X_{ikt}$ (k=1,..,K).

Objection to time period fixed effects: each time dummy has the same homogeneous impact on all observations in period t, while it is likely that, for example, business cycle effects hit one unit harder than another unit. Total number of common factor parameters to be estimated when accounting for heterogeneity by CSAs increases to N+(2+K)*N.

Since the numbers of parameters to be estimated increases rapidly with the number of common factors, most empirical studies try to keep the number of cross-sectional averages to a minimum. Often controlling for $\bar{Y}_t$ and $\bar{Y}_{t-1}$ only already effectively filters out the common time trends in the data.

Pesaran (2006, assumption 5 and remark 3): CSAs may be treated as exogenous explanatory variables since the contribution of each unit to the CSAs at a particular point in time goes to zero if N goes to infinity.

Link with cyclical sensitivity literature

Thirlwall (1966) and Brechling (1967) demonstrate that regional unemployment rates tend to move in tandem with the national unemployment rate, but within the common rises and falls over time, the extent to which a region's rate responds to changes in the national rate can be quite heterogeneous.

This implies that heterogeneity is considered in both the old cyclical sensitivity literature and in the modern CSA literature and thus that common factors can be embedded in the economic-theoretical literature on cyclical sensitivity.

Option 3: Principal components, in which case the *Γ* parameters represent the factor loadings of the principal components.

Shi and Lee (2017): Develop QML estimator for the dynamic GNS model with CF specified as principal components. This estimator does not require any specification of the distribution function of the disturbance term (explains the Q in QML). The coefficients estimates are bias-corrected for the Nickell bias and the impact of this bias on the other coefficients in the equation.

For this purpose, a Matlab routine called SFactors has been developed, which the first author (Shi) made available at his web site www.w-shi.net. I extended this with the calculation of R2 and the log-likelihood function value and posted this at spatial-panels.com.

A potential disadvantage of principal components is that they are often difficult to interpret, especially if they are compared with cross-sectional averages.

Every principal component requires the estimation of 2N additonal parameters.

Empirical Results CF

To find out which set of common factors is able to filter out common factors most effectively, the cross-sectional dependence (CD) test developed by Pesaran (2015) may be used.

The conclusion from three empirical studies — Cicarelli and Elhorst (2018), Elhorst et al. (2020) and Elhorst (2021) — is that the best option (1, 2 or 3) to control for common time trends might differ from one empirical study to another.

Conclusion

The general nesting spatial (GNS) econometric model for spatial panels with common factors (CF) is the most general spatial econometric model currently available for empirical research.

I encourage more scholars to work with this model in their empirical research.

At the same time, I should warn you that this is a difficult model to work with since the estimation results produced by this model are often quite puzzling, especially in the beginning.

This advanced model requires extensive research experience in spatial econometrics and sufficient economic-theoretical knowledge of the problem at hand. Often the results are not immediately in line with initial expectations, but after thinking them over and debating them with other researchers, progress towards an acceptable model specification can be made step by step.