

Econometric models for spatial panels with common factors

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9th International Conference on Applied Theory, Macro and Empirical Finance

April 14th - 15th

University of Macedonia, Thessaloniki, Greece

Download this presentation and a Matlab computer lab I use for teaching:

spatial-panels.com (scroll down on the home page and click on Thessaloniki2025)

See also Elhorst JP (2022) *Review of Regional Research*

<https://doi.org/10.1007/s10037-021-00163-w>

The purpose of this presentation is to encourage more researchers to use econometric models for spatial panels with common factors

A general econometric model for spatial panels with common factors

$$Y_t = \tau Y_{t-1} + \rho W Y_t + \eta W Y_{t-1} + X_t \beta + W X_t \theta + \sum_r \Gamma_r^T f_{rt} + u_t,$$

$$u_t = \lambda W u_t + \varepsilon_t$$

Spatial lags are in red: $1+K+1=K+2$ in total (K is number of X variables)

Dynamic effects are in green: 2 in total

Common factors are in blue: # parameters depend on type of CF, they are reported below.

In this presentation, the rationale behind each term, but also potential objections or pitfalls to incorporating it from a statistical or economic-theoretical point of view are explained. Reason: I want to encourage more researchers to start using this model, though only in a responsible way.

Model motivation based on pre-testing

Elhorst, Gross and Tereanu (2021) *Journal of Economic Surveys*

Many outcome variables are characterized by cross-sectional dependence (CSD), which can take two forms (Chudik and Pesaran 2011)

1. Local dependence → 'weak' CSD → relates to spatial models
2. Global dependence → 'strong' CSD → relates to common factors (GVARs)

Bailey, Holly, Pesaran (2016): two-step test procedure.

CD = $\sqrt{2T/N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\rho}_{ij}$ tests for strong vs. weak CSD

Exponent α computes the degree of CSD based on the speed of convergence of the average correlation coefficient, provided CSD is not weak

$$\overline{\rho_N} = \frac{2}{N(N-1)} \sum_{i=1}^N \sum_{j=i+1}^N \rho_{ij} = O(N^{2\alpha-2}), \text{ } \alpha=0.75 \text{ corresponds to } O(N^{-1/2})$$

Dominant outcome: $\alpha=1$ on the raw data in the first round and $\alpha<0.75$ on the residuals after controlling for CF in the second round. This outcome points to both common factors and local spatial dependence.

WY_t

A spatial lag in the dependent variable implies that y_{it} observed in cross-sectional unit i is explained by y_{jt} in other cross-sectional units $j, j \neq i$, and vice versa (two-way relationship).

The units j which are included depend on the specification of the spatial weight matrix W .

A linear regression model that contains a spatial lag in the dependent variable only (WY_t) is known as a spatial autoregressive (SAR) model. The SAR model is one of the most widely used spatial econometric models to introduce new methods of estimation or of spatial statistics

Note: Other models used for this purpose: SEM (Wu_t) and SAC=SARAR (WY_t, Wu_t).

Estimation (SAR model)

Ord (1975): maximum likelihood (ML) estimator.

Anselin (1988), Kelejian and Prucha (1998, 1999): instrumental variables (IV) and generalized method-of-moments (GMM) estimators + regularity conditions W .

Lee (2004): quasi maximum likelihood (QML) estimator + regularity conditions W .

LeSage and Pace (2009, Ch.5): Bayesian Markow Chain Monte Carlo (MCMC) estimator (improvement of LeSage, 1997).

Bao and Ullah (2007): finite sample properties of ML estimator.

Ahrens and Bhattachajee (2015): Lasso 2SLS estimator.

Kyriacou et al. (2017): Indirect Inference (II) estimator.

Smirnov (2021): closed-form consistent estimator based on ML.

Economic-theoretical underpinnings of WY_t

- Ertur and Koch (2007): SD model of GDP per capita growth (initial income level, savings rate, population growth rate).
- Anselin (2006): Conceptualization of strategic interaction or a spatial reaction function, $y_i = R(y_{-i}, x_i)$, where y_{-i} reflects decisions by other agents, among which local governments, petrol stations, etcetera.
- Blonigen et al (2007): foreign direct investments (FDI).
- Behrens et al. (2012): a quantity-based structural gravity equation system in which both trade flows and error terms are cross-sectionally correlated.
- Blume et al. (2015): Linear social interaction models.
- Xu and Lee (2019): SAR can be regarded as a model on the Nash equilibrium of a static complete information game with a linear-quadratic utility function.
- Lewbel et al. (2023): Social networks with unobservable links.
- Fernandez, V. (2009). Spatial linkages in international financial markets. Quantitative Finance, 11(2), 237–245.

Critique on including WY_t

Pinkse and Slade (2010): SAR model is fit onto every empirical problem. Entire spatial dependence structure is reduced to one single unknown coefficient.

McMillen (2012): SAR model (or SE model) is used as a quick fix for nearly any data set related to space.

Corrado and Fingleton (2012): WY_t variable picks up omitted WX_t variables or nonlinearities in the X variables.

Reduced-form equation:

$$Y_t = (I_N - \rho W) (\tau Y_{t-1} + \eta WY_{t-1} + X_t\beta + WX_t\theta + \sum_r \Gamma_r^T f_{rt} + u_t)$$

$$\text{Elhorst (2010): } \left[\frac{\partial E(Y_t)}{\partial X_{1kt}} \dots \frac{\partial E(Y_t)}{\partial X_{Nkt}} \right] = ((1 - \tau)I_N - (\rho + \eta)W)^{-1} (I_N\beta_k + W\theta_k).$$

Ratio indirect and direct effect=the same for every explanatory variable in SAR model.

WX_t Halleck Vega and Elhorst (2015)

1. SAR (WY_t), SE (Wu_t) and SAC=SARAR (WY_t, Wu_t) models are of limited use in empirical research due to initial restrictions on the spillover effects they can potentially produce. Ratio between indirect and direct effect is zero (SEM) or the same for every explanatory variable (SAR, SAC).
 2. Since there are K regressors WX_t , and only one WY_t and only one error term Wu_t , applied researchers better focus on WX_t variables first.
 3. W can easily be parameterized ($w_{ij} = 1/d^\gamma, w_{ij} = \exp[-\gamma d]$).
- Recently, Tan, Kesina and Elhorst (2025, Political Analysis) succeeded in parameterizing each spatial weight matrix of WY_t and the K spatial lags of WX_t with a different instead of one common distance decay parameter γ , which further increases the flexibility of spillover effects, as their spatial ranges may then also differ.
4. Since WX_t variables do not cause severe additional econometric problems (regularity conditions), endogeneity of X_t and WX_t variables can be tested for using standard (non-spatial) IV tests.

Economic-theoretical underpinning of WX_t variables

Ertur and Koch (2007): SD model of GDP per capita growth (initial income level, savings rate, population growth rate).

LeSage and Pace (2009): Several motivations, though mostly statistical.

Bramoullé et al. (2009): Identification of peer effects (and contextual effects).

Yesilyurt and Elhorst (2017): SD model of military expenditures as a ratio of GDP.

Firmino Costa da Silva et al. (2017): dynamic SD and GNS model of a spatially augmented population growth model.

Heijnen and Elhorst (2018): SD diffusion model of waste disposal taxes across municipalities.

Xu and Lee (2019): game-theoretical model can be extended with WX_t variables.

Wu_t

Determinants omitted from the model are spatially autocorrelated, or unobserved shocks follow a spatial pattern.

Wu_t affects efficiency but not the consistency of the parameter estimates.

Dynamics: Y_{t-1} and WY_{t-1}

Habit persistence. It takes time to change behavior.

Korniotis (2010): Internal (τ) and external (η) habit persistence.

Anselin et al. (2008): time-space recursive spatial econometric model (WY_t not included). Suitable to explain **spatial diffusion phenomena**. Think of the rise and spread of the Covid-19 virus on a daily basis.

LeSage and Pace (2009, ch. 7): spatiotemporal (partial adjustment) model. High temporal dependence and low spatial dependence might nonetheless imply a long-run equilibrium with high spatial dependence.

Goyal (2009, ch. 5): the social or spatial reaction function may take the form $y_{it} = R(y_{it-1}, y_{-it-1}, x_i)$, y_{-it-1} reflects decisions by other agents in the previous period.

Fogli and Veldkamp (2011): Information diffusion can change preferences, but because people need time to gather information, there is a delay in the decision-making process and the emergence of spatial dependency.

$$\sum_r \Gamma_r^T f_{rt}$$

Option 1: Two factors $f_{1t} = (1, \dots, 1)^T$ and $f_{2t} = (\xi_1, \dots, \xi_{T-1})^T$, with parameters of respectively $\Gamma_1^T = (v_1, \dots, v_N)$ and $\Gamma_2^T = (1, \dots, 1)$, gives a dynamic GNS model with **cross-sectional and time-period fixed effects**.

\mathbf{v} : vector of cross-sectional fixed effects ($i=1, \dots, N$)

ξ_t : time period fixed effects ($t=1, \dots, T-1$)

Number of CF parameters: $N+T-1$.

Option 2: Keep the cross-sectional fixed effects, but replace the time dummies by cross-sectional averages (CSAs) of the main variables (**read: not their spatial lags**) with **unit-specific coefficients**: $\bar{Y}_t = \frac{1}{N} \sum_{i=1}^N Y_{it}$, $\bar{Y}_{t-1} = \frac{1}{N} \sum_{i=1}^N Y_{it-1}$ and $\bar{X}_{kt} = \frac{1}{N} \sum_{i=1}^N X_{ikt}$ ($k=1, \dots, K$).

-Objection to time period fixed effects: each time dummy has the same homogeneous impact on all observations in period t , while it is likely that, for example, business cycle effects hit one unit harder than another unit. Total number of common factor parameters to be estimated when accounting for heterogeneity by CSAs increases to $N+(2+K)*N$.

-Since the numbers of parameters to be estimated increases rapidly with the number of common factors, most empirical studies try to keep the number of cross-sectional averages to a minimum. Often controlling for \bar{Y}_t and \bar{Y}_{t-1} only already effectively filters out the common time trends in the data.

-Pesaran (2006, assumption 5 and remark 3): CSAs may be treated as exogenous explanatory variables since the contribution of each unit to the CSAs at a particular point in time goes to zero if N goes to infinity.

Option 3: Principal components, in which case the Γ parameters represent the factor loadings of the principal components.

Every principal component requires the estimation of $2N$ additional parameters.

Note: Two principal components are needed to replace cross-sectional and time-period specific effects.

Shi and Lee (2017) and Bai and Li (2021) develop QML estimators and Matlab code for dynamic spatial econometric models with CF specified as principal components.

Cui et al. (2023) and Kripfganz and Sarafidis (2025) develop IV estimators and Stata code for dynamic spatial econometric models with CF specified as principal components.

A potential disadvantage of principal components is that they are often difficult to interpret, especially if they are compared with cross-sectional averages.

Empirical Results CF

The conclusion from three empirical studies — Cicarelli and Elhorst (2018) on cigarette demand in Italian regions 19th century, Elhorst et al. (2020) on car use in French departments and Elhorst (2021) on cigarette demand in US states — is that the best option (1, 2 or 3) to control for common time trends might differ from one empirical study to another.

Empirical illustration taken from Kripfganz and Sarafidis (Stata journal, spxtivdfreg)

Dependent variable NPL = ratio of non-performing loans to total loans of 350 US banking institutions over the period 2006:Q1-2014:Q4.

	Full model	Without factors	Without spatial lag
$\hat{\psi}$ ($\mathbf{W}_N \text{ NPL}_t$)	0.394*** (0.085)	0.288*** (0.038)	
$\hat{\rho}$ (NPL_{t-1})	0.290*** (0.054)	0.594*** (0.034)	0.323*** (0.055)
$\hat{\beta}_1$ (INEFF_t)	0.447*** (0.105)	0.366*** (0.107)	0.638*** (0.116)
$\hat{\beta}_2$ (CAR_t)	0.031*** (0.006)	0.017*** (0.004)	0.030*** (0.006)
$\hat{\beta}_3$ (SIZE_t)	0.223** (0.094)	0.089 (0.061)	0.346*** (0.096)
$\hat{\beta}_4$ (BUFFER_t)	-0.055*** (0.012)	-0.025** (0.010)	-0.045*** (0.016)
$\hat{\beta}_5$ (PROFIT_t)	-0.005*** (0.002)	-0.006*** (0.002)	-0.004** (0.002)
$\hat{\beta}_6$ (QUALITY_t)	0.183*** (0.031)	0.283*** (0.029)	0.183*** (0.036)
$\hat{\beta}_7$ (LIQUIDITY_t)	2.452*** (0.270)	0.843*** (0.180)	2.534*** (0.311)
\hat{r}_x	2	0	2
\hat{r}_y	1	0	1
J-test	18.825 [0.468]	48.151 [0.000]	8.174 [0.226]

Table 2: Coefficient estimates; see Section 4.1 for details on the model specification.

Long-run impacts

		Delta-method				
		Impact	std. err.	z	P> z	[95% conf. interval]
direct						
	INEFF	.6470588	.1593924	4.06	0.000	.3346554 .9594623
	CAR	.0441245	.0092325	4.78	0.000	.0260292 .0622198
	SIZE	.3219497	.1416728	2.27	0.023	.044276 .5996233
	BUFFER	-.0788324	.0183176	-4.30	0.000	-.1147342 -.0429306
	PROFIT	-.0077164	.0023773	-3.25	0.001	-.0123757 -.003057
	QUALITY	.2647392	.0466629	5.67	0.000	.1732816 .3561968
	LIQUIDITY	3.546983	.4454284	7.96	0.000	2.673959 4.420007
indirect						
	INEFF	.7694677	.3352809	2.29	0.022	.1123291 1.426606
	CAR	.0524719	.0237326	2.21	0.027	.0059569 .0989868
	SIZE	.3828552	.1975749	1.94	0.053	-.0043845 .770095
	BUFFER	-.0937457	.0428643	-2.19	0.029	-.1777581 -.0097333
	PROFIT	-.0091761	.0046348	-1.98	0.048	-.0182603 -.000092
	QUALITY	.3148218	.1408165	2.24	0.025	.0388266 .590817
	LIQUIDITY	4.217992	1.742264	2.42	0.015	.8032163 7.632767

The dynamic model components can be specified with the following options:

- `splag` requests to include a spatial lag of the dependent variable as an additional regressor; i.e., $\sum_{j=1}^N w_{ij}y_{jt}$;
- `tlags(#)` requests to include `#` time lags of the dependent variable as additional regressors; i.e., $y_{it-1}, \dots, y_{it-#}$;
- `sptlags(#)` requests to include `#` spatial time lags of the dependent variable as additional regressors; i.e., $\sum_{j=1}^N w_{ij}y_{jt-1}, \dots, \sum_{j=1}^N w_{ij}y_{jt-#}$;
- `spindepvars(varlist)` requests to include spatial lags of the specified variable list as additional regressors; i.e., $\sum_{j=1}^N w_{ij}\mathbf{x}_{jt}^\top$.

Conclusion

I encourage more scholars to work with econometric model for spatial panels with common factors in their empirical research.

At the same time, I should warn you that this is a difficult model to work with since the estimation results produced by this model are often quite puzzling, especially in the beginning.

This advanced model requires extensive research experience in spatial econometrics and sufficient economic-theoretical knowledge of the problem at hand. Often the results are not immediately in line with initial expectations, but after thinking them over and debating them with other researchers, progress towards an acceptable model specification can be made step by step.